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Wave Runup on Variable Beach Profiles

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Introduction

A common problem faced by practicing engineers is the determination of wave runup on coastal structures and beaches having nonuniform and often complex slope characteristics. For shore-line applications the calculation and prediction of wave runup along the open coast is of practical interest to those engaged in the management and design aspects of the coastal zone. The determination of the limits of wave uprush provides valuable input to the evaluation of a wide range of coastal works projects including the design and placement of shore protection structures and beach fill, the assessment of storm damages, and the implementation of major resource management programs such as the Florida Coastal Construction Set-Back Line (8,16). In view of the importance of these types of analyses, it is somewhat surprising that relatively little work has been done that directly addresses the problem of wave runup on variable-sloped beach profiles. This paper presents one possible method for the prediction of wave runup on beaches. It is hoped that this method, though not yet verified, will stimulate interest in the coastal engineering community.

Saville's Composite Slope Method

The runup of breaking waves on uniform slopes has been examined by numerous investigators including Saville (10), Savage (9), Van Dorn (15), and Granthem

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(4). A comprehensive review of existing theoretical and experimental works has been provided by Le Méhauté, et al. (7), who conclude that experimental results provide the best available treatment of runup for breaking waves.

For composite or variable-slope geometries, however, very little work has been done. As an alternative to the costly and time-consuming procedure of scale-model testing, Saville (11) showed that experimental results for uniform slopes could be used to reasonably predict runup on composite-slope structures. Since its first appearance in the literature, Saville's composite-slope method has gained wide acceptance and is included in the US Army Shore Protection Manual (12).

The use of Saville's method of composite slopes to calculate wave runup on beaches has both advantages and disadvantages. Principal advantages include: (1) The ability to represent irregular features such as offshore bars; and (2) the flexibility of changing the number of linear segments used to achieve a better representation of the profile geometry. Disadvantages of this method include the following: (1) Computation of a mean composite slope is cumbersome due to the piecewise continuous representation used for the profile geometry; (2) singularities are introduced into the profile schematization at each juncture of the linear segments used; and (3) variations in the stillwater level are not easily accounted for.

The method presented herein eliminates these disadvantages by using a smooth functional representation of the profile geometry rather than the piecewise linear segmentation proposed by Saville. The functional form used is not presently capable of representing irregular features such as offshore bars and other perturbations in the profile geometry. However, this disadvantage can be overcome by the selection of alternative profile functions. The general approach described herein would still apply.

Representation of Beach Profile Geometry

The selection of a particular functional form to represent the profile geometry is somewhat arbitrary, provided that satisfactory agreement is achieved between the functional approximation and the actual profile. Everts (3) used NOS 1200 series chart data along 441 profile lines from New York to Texas to show that the "shore face" or nearshore region of the continental shelf approximates a concave upward exponential curve. Dean (2), using theoretical arguments, examined three plausible mechanisms that could govern the form of beach profiles in the surf zone: (1) Uniform average longshore shear stress; (2) uniform average wave energy dissipation rate per unit plan area; and (3) uniform average wave energy dissipation rate per unit water volume. He showed that all three of these mechanisms produce an equilibrium profile geometry of the form

$$d = As^m \qquad (1)$$

in which d = the water depth; s = the horizontal distance from shore; A = a scale factor; and m = a shape factor. Eq. 1 is in general agreement with the work of Bruun (1) who developed a similar expression from the analysis of beach profiles along the Danish and California coasts.

Based upon the work of Dean and Bruun a power curve similar to that given by Eq. (1) was selected. The profile geometry is described by a parabola of the form

$$x = ay^{\nu} \qquad (2)$$

as shown in Fig. 1. In this figure the coordinate axes have their origin located at the dune crest with x positive in the seaward direction and y positive downward. The vertical distance from the dune crest to mean sea level (MSL) is denoted by y_0. As shown, R = the wave runup distance above the stillwater level; d_b = the wave breaking depth; and η = the surge elevation with respect to MSL. In describing the profile geometry, parameter a is analogous to the scale factor, and ν = the shape factor as described by Dean (2).

The formulation of the wave-runup problem involves a total of 11 variables, i.e., H_0', H_b, d_b, L, T, $\cot\theta$, R, η, y_0, a, and ν. The solution technique used reduces the number of unknowns to five by specifying: (1) The profile geometry (y_0, a, ν); (2) deepwater wave height H_0'; (3) wave period T; and

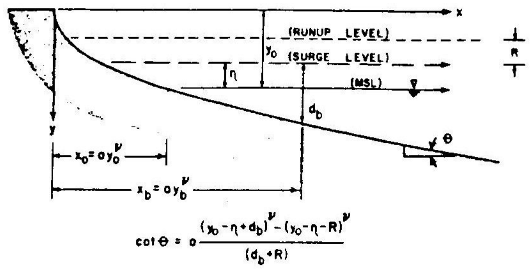

FIG. 1.—Definition Sketch

(4) surge elevation η. Relationships for the remaining unknowns are obtained as follows.

From linear wave theory the effect of shoaling on wave height is given by

$$\frac{H}{H_0'} = \left[\left(1 + \frac{\dfrac{4\pi d}{L}}{\sinh\dfrac{4\pi d}{L}}\right)\tanh\frac{2\pi d}{L}\right]^{-1/2} \qquad (3)$$

in which H_0' represents the deepwater wave-height equivalent of the shallow water wave height if shoaling effects, only, are considered. The corresponding expression for wave length L is

$$L = \frac{gT^2}{2\pi}\tanh\frac{2\pi d}{L} \qquad (4)$$

in which T = the wave period; and d = the water depth.

The mean slope of the profile section lying between the wave break point, and the limit of wave uprush is given by

$$\overline{\cot\theta} = \frac{1}{d_b + R}\int_{y_0-\eta-R}^{y_0-\eta+d_b} \cot\theta \, dy \qquad (5)$$

From Eq. 2 $\cot\theta = \dfrac{dx}{dy} = a\nu y^{\nu-1}$ . . . . (6)

Substituting Eq. 6 into Eq. 5 and carrying out the integration yields

$$\overline{\cot\theta} = \frac{a}{d_b + R}\left[(y_0 - \eta + d_b)^{\nu} - (y_0 - \eta - R)^{\nu}\right] \quad (7)$$

which represents the mean bottom slope between the wave break point and the limit of wave uprush.

To obtain a relationship for the wave runup as a function of the profile geometry and wave characteristics, the experimental results of Saville, as presented in Fig. 7-11 of Ref. 9, are reduced to equation form. For breaking waves these results are reasonably well behaved and can be expressed as

$$\frac{R}{H_0'} = K\left(\frac{H_0'}{gT^2}\right)^{n_1}(\cot\theta)^{n_2} \quad (8)$$

A Comparison of Eq. 8 with Saville's results yields values of $K = 0.42$, $n_1 = -0.45$, and $n_2 = -1.02$. Assuming $n_1 \simeq -0.5$ and $n_2 \simeq 1$, Eq. 8 can then be written as

$$\frac{R}{H_0'} = 0.42\sqrt{\frac{gT^2}{H_0'}}\tan\theta \quad (9)$$

which agrees well with the results of Hunt (6). Support for the use of empirical relationships such as Eq. 9 is provided by Le Méhauté, et al., who found good agreement between predicted values using Hunt's relationship and experimental data summarized in Ref. 13. To verify this a series of calculations was performed using Eq. 9, and the results were compared with corresponding values obtained from Fig. 7-11 of Ref. (12). For $H_0'/(gT^2) \geq 0.0016$ agreement within $\pm 10\%$ was obtained for mean slope values ranging from 0.038 to 0.29. For lower steepness waves, $0.003 \leq H_0'/(gT^2) < 0.0016$, agreement within $\pm 10\%$ was obtained for $0.083 \leq \tan\theta \leq 0.25$.

Eqs. 3, 4, 7, and 9 are sufficient for the determination of wave runup values provided that the relationship between wave height and water depth at breaking is known. This relationship can be expressed as

$$H_b = \kappa d_b \quad (10)$$

in which κ represents the ratio of breaking wave height to water depth at breaking. An exact relationship between H_b and d_b is not presently well defined in the literature. However, it is known that κ varies with changes in bottom slope and incident wave steepness. In general, increasing wave steepness with constant bottom slope causes κ to decrease whereas increasing bottom slope with fixed wave steepness causes κ to increase. For most beach applications κ is expected to fall within the range $0.7 \leq \kappa \leq 1.4$. In the work presented herein, it is assumed that $\kappa = 1$, which yields the simple relationship of

$$H_b = d_b \quad (11)$$

The use of κ values greater than one will produce wave runup values that

are consistently higher than those presented later in this paper. Similarly, κ values less than one will produce consistently lower values of runup. The amount by which the calculated runup varies for these cases is dependent upon the particular surge elevation and profile geometry used.

Through the use of Eq. 11 the shallow-water wave height, H, can now be eliminated from Eq. 3 by evaluating this expression at the wave breakpoint and substituting d_b for H_b, or

$$d_b = H_0'\left[\left(1 + \frac{\dfrac{4\pi d_b}{L}}{\sinh\dfrac{4\pi d_b}{L}}\right)\tanh\frac{2\pi d_b}{L}\right]^{-1/2} \quad (12)$$

Eqs. 4, 7, 9, and 12 provide the necessary relationships for the determination of wave runup values. These expressions are further simplified by the introduction of the following nondimensional parameters

$$T' = \frac{gT^2}{2\pi y_0} \quad (13)$$

$$S' = \frac{\eta}{y_0} \quad (14)$$

$$H' = \frac{H_0'}{y_0} \quad (15)$$

$$R' = \frac{R}{y_0} \quad (16)$$

$$D' = \frac{d_b}{y_0} \quad (17)$$

$$L' = \frac{d_b}{L} \quad (18)$$

Combining Eqs. 13–18 with Eqs. 4, 7, 9, and 12 yields the corresponding nondimensional relationships

$$\overline{\cot\theta} = \frac{a y_0^{\nu-1}}{D' + R'}\left[(1 - S' + D')^{\nu} - (1 - S' - R')^{\nu}\right] \quad (19)$$

$$R' = \frac{0.42}{\overline{\cot\theta}}\sqrt{2\pi H' T'} \quad (20)$$

$$D' = L'T'\tanh 2\pi L' \quad (21)$$

$$D' = H'\left[\left(1 + \frac{4\pi L'}{\sinh 4\pi L'}\right)\tanh 2\pi L'\right]^{-1/2} \quad (22)$$

The procedure used to solve Eqs. 19–22 is suitable for solution by computer and provides a series of curves that describes the wave-runup behavior on a given profile geometry as a function of deepwater wave height and surge elevation. These curves can be developed for varying values of wave period. In carrying

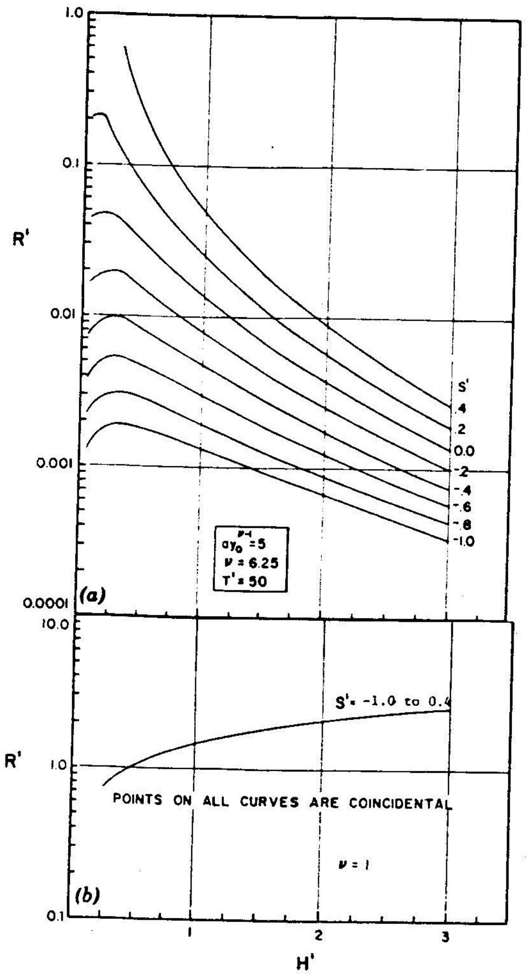

FIG. 2.—Wave Runup on: (a) Variable Slope Profile; (b) Profile of Uniform Slope

out the analysis, the effects of wave refraction, diffraction, and bottom friction are ignored.

Example Calculations

The solution technique used is similar to that proposed by Saville (11). To begin, values for T', H', and S' are specified. Eqs. 21 and 22 are then solved simultaneously for D' and L' using a standard Newton Raphson technique (see Ref. 5). Knowing D', S', and the profile geometry parameters a, y_0, and ν, Eq. 19 is solved for $\overline{\cot\theta}$ using an arbitrary initial estimate of the nondimensional runup, R'. The value obtained for $\overline{\cot\theta}$ is then used in Eq. 20 to yield an improved value for R', and the solution of Eqs. 19 and 20 is repeated until the successive values calculated for $\overline{\cot\theta}$ and R' converge to the final solution.

To illustrate some typical results obtained using this method, six sets of calculations were carried out. The first set of calculations was performed for an arbitrary profile having a steep beach and dune face with mild offshore slopes. Parameter values of $ay_0^{\nu-1} = 5$, $\nu = 6.25$, and $T' = 50$ were used. Because the profile geometry selected for this particular example is arbitrary, unique values of a, y_0, and T do not exist. For actual profile geometries, as will be shown later, this is not the case. Fig. 2(a) shows the calculated values of R' as a function of the nondimensional wave height, H', and surge, S'. Values of $(R' + S') \geq 1$ correspond to overtopping conditions and are not

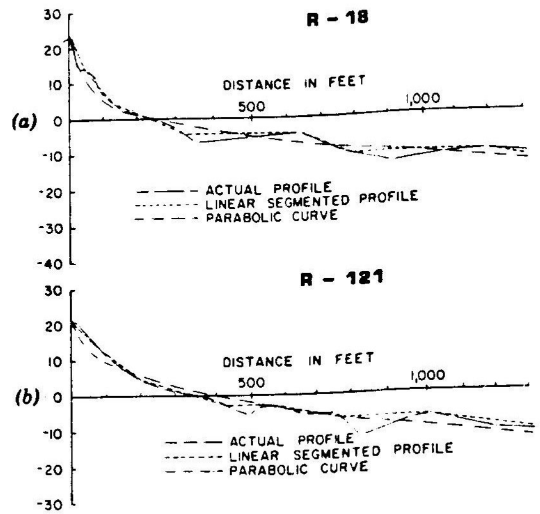

FIG. 3.—Volusia County Profiles with Linear Segmented and Parabolic Approximations: (a) R-18; (b) R-121 (1 ft = 0.305 m)

shown. The curves do show, however, the effects of profile geometry and surve elevation on wave runup. For fixed values of H', increases in the surge elevation produce higher values of runup. By increasing S', waves of specified height are allowed to break closer to shore thereby exposing a steeper mean slope between the break point and the limit of wave uprush. The steeper mean slope produces, in turn, a higher value of runup.

The effects of mean slope and wave height on runup are best seen from Eq. 20. For fixed T' this expression may be written as

$$R' \alpha H'^{1/2} \overline{\tan\theta} \quad (23)$$

This functional dependence is demonstrated clearly by the curves shown in Fig. 2(a). For fixed S', the calculated runup is seen to increase, reach a maximum, and then decrease with increasing H'. Obviously, within the region of increasing

R' with increasing H', the runup is governed predominantly by changes in wave height. However, the peaking and subsequent decrease in R' with increasing H' is influenced primarily by the reduction in mean slope as the wave break point moves farther and farther offshore.

The shape of the curves is strongly influenced by the particular profile geometry used. More specifically, variations in the shape factor, ν, affect the relative

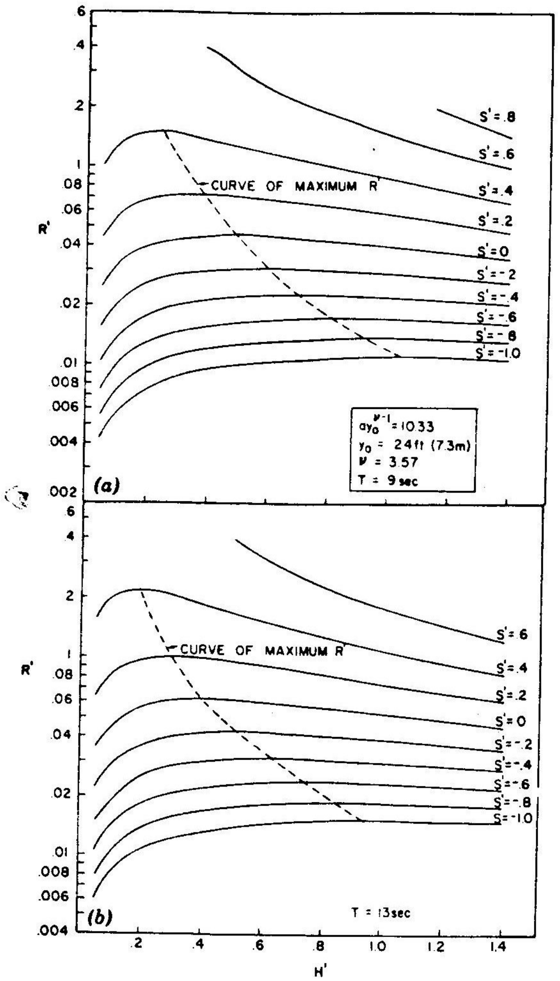

FIG. 4.—Wave Runup on Profile R-18: (a) T = 9 sec; (b) T = 13 sec

importance of wave height and mean slope on the calculated runup. Smaller values of ν tend to suppress the influence of slope on runup with respect to wave height, whereas large values of ν produce the opposite effect. Increases in surge elevation tend also to emphasize the influence of mean slope on the calculated runup. These trends can be observed by comparing Figs. 2, 4, and 5.

To illustrate the wave runup on uniform slopes, a series of calculations was performed for ν = 1. Values used for all other parameters remain the same as those described for the previous example. Results are shown in Fig. 2(*b*). As expected, variations in surge elevation have no effect on wave runup for the uniform slope case. Moreover, increases in wave height consistently produce higher values of runup. This behavior is in agreement with that predicted by Eq. 23 with $\tan \theta$ = constant.

The remaining calculations were performed using beach and offshore profile data obtained by the Florida Department of Natural Resources. The two profiles selected are located along the coast of Volusia County, Fla. These are designated R-18 and R-121 and are shown in Figs. 3(*a*) and 3(*b*). Each figure presents

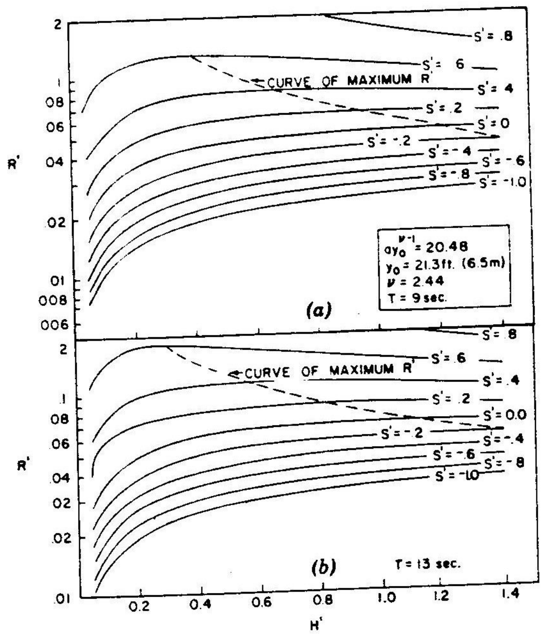

FIG. 5.—Wave Runup on Profile R-121: (a) T = 9 sec; (b) T = 13 sec

three representations of the profile geometry, i.e., a plot of the actual profile data, a linear segmented representation for use in Saville's composite slope method, and a least-squares fit parabolic curve of the form given by Eq. 2. Values of y_0 for each profile were obtained directly from the profile plots as the elevation of the primary dune crest above MSL. Values for a and ν were obtained from the least-squares curve fit of Eq. 2 to the actual profile geometry.

Runup calculations were carried out for each profile using wave periods of 9 sec and 13 sec. Figs. 4(*a*) and 4(*b*) show the calculated results for profile R-18, while Figs. 5(*a*) and 5(*b*) show the corresponding results for profile R-121. Parameter values used in each series of calculations are shown in the appropriate figures. The loci of maximum wave runup are shown by the dotted curves.

A comparison of the curves shown in Figs. 4(*a*), 4(*b*), 5(*a*), and 5(*b*) with those shown in Fig. 2(*a*) clearly shows the influence of profile geometry on the calculated wave runup. The smaller values of ν used for the Volusia County profiles produce a flatter response of the runup to changes in wave height. Moreover, the range of deepwater wave heights that produce the maximum value of runup is increased considerably. This change in behavior is particularly noticeable for profile R-121 [Figs. 5(*a*) and 5(*b*)]. For the range of H' values examined and $S' \leq 0$, the runup on this profile behaves in a manner very similar to that presented for the uniform slope case [Fig. 2(*b*)]. This same effect is seen to a lesser degree in Figs. 5 and 6 for profile R-18.

The effect of increasing wave period on runup can be examined by the comparison of Fig. 4(*a*) with Fig. 4(*b*) and Fig. 5(*a*) with Fig. 5(*b*). Increasing T from 9 sec to 13 sec for the cases examined causes a nearly uniform upward shift in the curves with almost no change in shape.

Comparison of Results

Profiles R-18 and R-121 shown in Figs. 3(*a*) and 3(*b*) were used by Taylor, et al. (14) to calculate wave runup using Saville's method of composite slopes. In carrying out their analysis the wave period was fixed at $T = 13$ sec, and values of runup were calculated for varying surge elevations and breaking wave heights. The linear segmented approximations of the actual profile geometries used by these investigators are also shown in Figs. 3(*a*) and 3(*b*).

For the purpose of comparison, the conditions examined by Taylor, et al. were recalculated using the least-squares fit parabolic profiles previously described for R-18 and R-121. Figs. 6(*a*) and 6(*b*) show the results of this comparison. The dotted curves describe the runup behavior using a parabolic representation of the profile geometry whereas the results obtained by Taylor, et al. are shown by the solid curves.

Agreement between the two methods is generally poor. In fact, significant differences in the calculated behavior of the wave runup are apparent. These are as follows: (1) The parabolic profile method produces a much flatter response in the wave runup for varying H'; (2) for fixed values of S' runup values obtained using Saville's method are generally higher than those obtained using the parabolic profile method for the same conditions of surge elevation and wave height; and (3) for increasing S' the maximum wave runup predicted by the parabolic profile method increases with decreasing H'. The opposite trend is exhibited by values determined from Saville's method [Fig. 6(*b*)], except for some cases in which the trend suddenly reverses itself [e.g., Fig. 6(*a*)].

Differences 1 and 2 just mentioned can be attributed to the methods by which the actual profile geometry is approximated. The flatter response of the wave-runup behavior determined by the parabolic profile method is a result of the continuous smooth variations in profile slope and breaking water depth. Thus, the parabolic profile produces small changes in $\tan\theta$, D', and R' for small changes in H'. The corresponding changes obtained from Saville's method, however, have a tendency to be greater due to the limited number of linear segments used.

The tendency for Saville's method to predict higher values of R' can be explained by the different techniques used to represent the profile geometry and the resulting location of the wave breakpoint. Both profiles contain offshore bar features with negative slope segments extending 200 ft–300 ft (61 m–91.5 m) shoreward from the bar crest. In their investigation, Taylor, et al. represented these features by zero slope segments. Moreover, to provide conservative estimates of runup they assumed that a wave whose height equalled the depth of water on the shore face of the bar would break at the extreme shoreward limit of the zero slope segment. This assumption effectively steepens the profile geometry and results in a higher predicted value of wave runup. The parabolic-

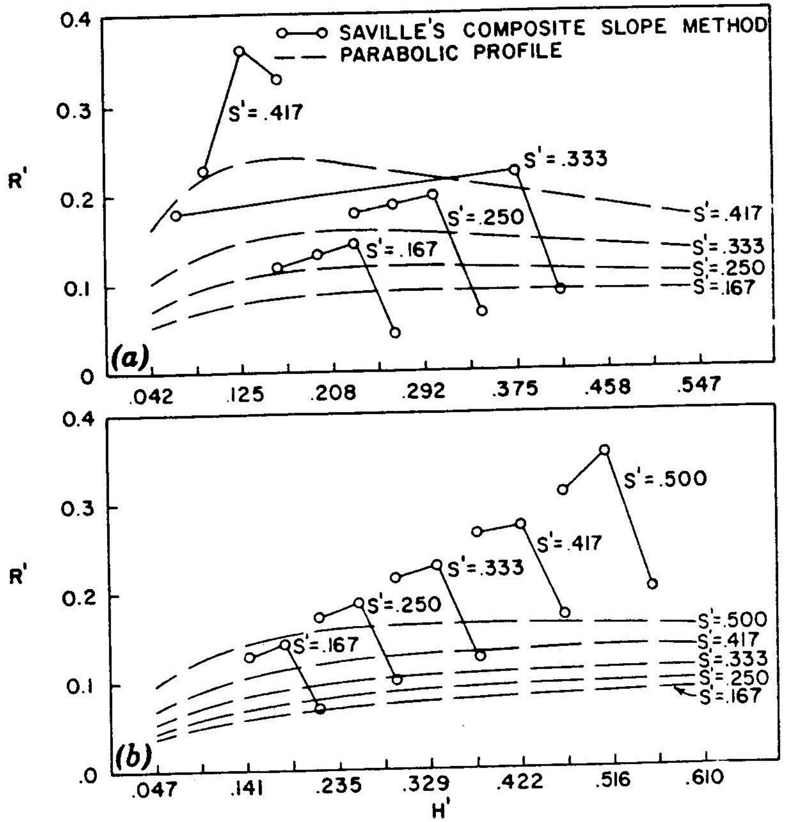

FIG. 6.—Comparison of Parabolic Profile Method Results with Composite Slope Method Results: (*a*) Profile R-18; (*b*) Profile R-121

profile method, however, passes a smoothly varying slope through the bar feature that typically intersects the zero slope segment seaward of its shoreward limit. This causes the corresponding wave break point to lie further offshore from that used by Taylor, et al., thereby reducing the mean slope and producing corresponding lower values of runup.

The apparent differences in behavior of calculated runup maxima as a function of H' and S' are not easily explained based upon the limited number of cases examined. From earlier analyses, it would appear that the maximum wave runup for the parabolic profile method is influenced primarily by changes in mean slope caused by shoreward translation in the wave break point due to increasing surge elevations. The fact that this does not occur for Saville's method could be explained by the manner in which Taylor, et al. approximated the offshore bar features by zero sloped segments. This causes the break point of the wave producing the maximum runup to remain fixed at the shoreward terminus of the horizontal segment for varying surge elevations. Thus, the behavior of the maximum runup is now influenced primarily by increasing H' with increasing S'. This behavioral trend continues until such time when, due to increasing surge, the influence of the bar feature is reduced and smaller waves of sufficient height are allowed to break upon the steeper portions of the profile. This causes a shift in the maximum runup point with respect to H'.

Summary and Conclusions

A method has been presented for the analysis of runup due to breaking waves on variable slope profiles using a parabolic representation of the profile geometry. The description of the profile geometry by a smooth function and the determination of a functional relationship for wave runup makes the problem suitable for solution on a computer. This provides an efficient means for determining the runup resulting from many varying conditions.

The description of the profile by a parabolic form has also provided a better understanding of the individual effects of wave height, surge elevation, and profile slope on runup.

A comparison of the results obtained by the composite-slope method and the parabolic-profile method when applied to two beach profiles along the coast of Volusia County, Fla., indicated that the computed runup is quite sensitive to the manner in which the profile geometry is described. The differences in the results produced by these two methods could all be explained in terms of the way each representation of the beach profile directly affected runup or by the influence that a particular profile representation exerted on other parameters affecting runup.

While it would be beneficial to identify which, if either, method more accurately predicted runup, it is not possible at this time since neither method has been verified.

Appendix I.—References

1. Bruun, P., "Coast Erosion and the Development of Beach Profiles," *Technical Memo No. 44*, Beach Erosion Board, United States Army Corps of Engineers, Washington, D.C., 1954.
2. Dean, R. B., "Equilibrium Beach Profiles," *Ocean Engineering Technical Memo No. 12*, Department of Civil Engineering and College of Marine Studies, University of Delaware, Newark, Del., Jan., 1977
3. Everts, C. H., "Geometry of Profiles Across Inner Continental Shelves of the Atlantic and Gulf Coasts of the United States," *Technical Paper No. 78-4*, United States Army Coastal Engineering Research Center, Washington, D.C., Apr., 1978.
4. Granthem, K. N., "Wave Run-up on Sloping Structures," *Transactions*, American Geophysical Union, Vol. 34, No. 5, 1953.
5. Hildebrand, F. B., *Advanced Calculus for Applications*, Prentice-Hall Inc., Englewood Cliffs, N.J., 1962.

6. Hunt, I. A., "Design of Seawalls and Breakwaters," *Journal of the Waterways and Harbors Division*, ASCE, Vol. 85, No. WW3, Proc. Paper 2172, Sept., 1959, pp. 123–152.
7. Le Méhauté, B., Koh, R. C. Y., and Hwang, L. S., "A Synthesis on Wave Run-up," *Journal of the Waterways and Harbors Division*, ASCE, Vol. 94, No. WW1, Proc. Paper 5807, Feb., 1968, pp. 77–92.
8. Purpura, J. A., "Establishment of a Coastal Setback Line in Florida," *Thirteenth Coastal Engineering Conference*, ASCE, 1972, pp. 1599–1615.
9. Savage, R. P., "Wave Runup on Roughened and Permeable Slopes," *Journal of the Waterways and Harbors Division*, ASCE Vol. 84, No. WW3, Proc. Paper 1640, May, 1958, pp. 1640-1–1640-38.
10. Saville, T., Jr., "Wave Runup on Shore Structures," *Journal of the Waterways and Harbors Division*, ASCE, Vol. 82, No. WW2, Proc. Paper 925, Apr., 1956, pp. 925-1–925-14.
11. Saville, T., Jr., "Wave Runup on Composite Slopes," *Proceedings of the Sixth Conference on Coastal Engineering*, ASCE, 1958.
12. *Shore Protection Manual*, Vol. 2, 2nd ed., United States Army Coastal Engineering Research Center, United States Government Printing Office, Washington, D.C., 1975.
13. "Shore Protection, Planning and Design," *Technical Report No. 4*, Beach Erosion Board, United States Army Corps of Engineers, Washington, D.C., 1961.
14. Taylor, R. B., Chen, M., Özsoy, E., and Lee, K., "Hurricane Surge and Wave Runup, Volusia County, Florida," *Report No. TC891-02*, Tetra Tech, Inc., Jacksonville, Fla., 1978.
15. Van Dorn, W. G., "Theoretical and Experimental Study of Wave Enhancement and Runup on Uniformly Sloping Impermeable Beaches," *Technical Report No. SIO 66-11*, University of California, Scripps Institute of Oceanography, May 1966.
16. Wang, F. C., Purpura, J. A., and Chiu, T. Y., "Computer Algorithm of Wave Runup on Beaches," *Proceedings of the Fourteenth Coastal Engineering Conference*, ASCE, 1974, pp. 1956–1975.

Appendix II.—Notation

The following symbols are used in this paper:

A = Dean's scale factor;
a = scale factor;
d = water depth;
d_b = water depth at wave breaking;
D' = d_b/y_0 = nondimensionalized water depth at wave breaking;
g = gravitational constant;
H = wave height;
H_b = wave height at breaking;
H' = H'_0/y_0 = nondimensionalized deep water wave height;
H'_0 = deep water wave height;
K = proportionality factor;
L = wave length;
L' = d_b/L = nondimensionalized water depth at wave breaking;
m = Dean's shape factor;
n_1 = arbitrary exponent;
n_2 = arbitrary exponent;
R = wave runup distance above stillwater level;
R' = R/y_0 = nondimensionalized wave runup;
S' = η/y_0 = nondimensionalized surge elevation;

s = horizontal distance seaward from shore line;
T = wave period;
T' = $(gT^2)/(2\pi y_0)$ = nondimensionalized wave period;
x = horizontal distance seaward from dune crest;
x_b = horizontal distance from dune crest to breaker line;
x_0 = horizontal distance from dune crest to stillwater shore line;
y = vertical distance downward from dune crest;
y_b = vertical distance from dune crest to existing stillwater level;
y_0 = vertical distance from dune crest to MSL;
η = surge elevation with respect to MSL;
θ = angle between profile bottom and horizontal datum;
κ = ratio of breaking wave height to breaking depth;
ν = shape factor; and
π = 3.1415926.