QUASIGEOSTROPHIC MODEL DEVELOPMENT FOR MULTIPLY CONNECTED DOMAINS AND APPLICATIONS TO THE LEVANTINE BASIN CIRCULATION

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The Harvard quasigeostrophic model has been generalized to include multiply connected domains as well as multiple segments of coasts and islands in either enclosed, semi-enclosed and open boundaries. The methodology ensures a unique solution to the initial boundary and consistency conditions required to conserve mass and energy budgets in the case of enclosed domains. The algorithm to implement the methodology is an extension of that developed by Milliff (1989), with a modification to the capacitance matrix method to include a required term in the circulation integrals, and an additional condition for adjustment of the mean fields to insure uniqueness of the solution.

The model results have been compared with the natural mode oscillations in multiply connected domain studied by Flierl (1977). The test cases consisted of the multiply connected circular basin with a circular island and semi-open domain derived from the same solution. In the case of a closed domain, the model is sensitive to the accuracy of the vorticity budgets specified in the initial conditions. Care taken to achieve the accuracy of the initial conditions with a given discretization scheme ensures overall accuracy of the model results.

Real data have been used to initialize the model and to perform quasigeostrophic calculations of the circulation in the Levantine Basin, a multiply connected, semi-open domain of the Eastern Mediterranean enclosing the island of Cyprus. Preliminary results indicate stable basin-wide circulation features which evolve in long time scales (several months) in agreement with the characteristics previously inferred from the POEM experiments. Test runs with different geographical selections of the open boundaries indicate the robustness of the model behaviour with respect to the location of the open boundary segments.

A CONSISTENT BAROCLINIC QUASIGEOSTROPHIC OCEAN MODEL AND APPLICATION TO MULTIPLY CONNECTED OCEAN DOMAINS

E. CASEY, C. LOZANO, R. MILLIFF, A.R. ROBINSON

work pursuant to developments by:

McWilliams (1977)
Pinardi and Milliff (1989)
Milliff (1989)

Acknowledgements:

Harvard University, UNESCO, CIES (Fullbright Comission

[redacted]

PE (Boussinesq)

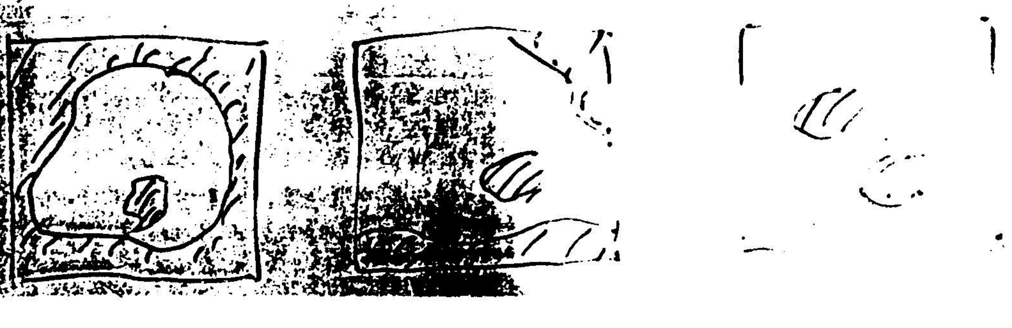
$$\left\{\begin{aligned}
&\epsilon \frac{D\vec{u}}{Dt} + (1+\epsilon\beta y)\,\hat{k}\times\vec{u} = -\nabla p + \epsilon \vec{f} \\
&p_z - b = 0 \\
&\nabla\cdot\vec{u} + \varepsilon w_z = 0 \\
&\varepsilon \frac{Db}{Dt} + \varepsilon\left[\Gamma^2\sigma(z)\right]^{-1} w = \varepsilon d \\
&+ \text{no-slip} + \text{flux boundary conditions}
\end{aligned}\right.$$

versus

QG

$$\left\{\begin{aligned}
&\frac{DQ}{Dt} = \hat{k}\cdot\nabla\times\vec{f} + (\Gamma^2\sigma d)_z \\
&\text{where } Q = \nabla^2\psi + (\Gamma^2\sigma\psi_z)_z + \beta y \\
&\vec{u}^{(0)} = \hat{k}\times\nabla\psi \\
&+ \text{qg boundary conditions}
\end{aligned}\right.$$

in enclosed, semi-enclosed or open domains, neighboring coastal boundaries.



Consistency Conditions:

Domain D

Domain Boundary:

$T = C_0 + C_1 + B_1 - B_1$

T' = simply connected boundary

D' = domain enclosed

Vorticity eqn:

$$\frac{\partial Q}{\partial t} + \alpha \nabla\cdot(Q\vec{u}^{(0)}) = \hat{k}\cdot\nabla\times\vec{f} + (\Gamma^2\sigma d)_z$$

$\downarrow$ Integrate over D':

$\downarrow$ use divergence theorem ....

$$-\oint_{T'} \vec{u}^{(1)}\cdot\hat{n}\,dl = \frac{\partial}{\partial t}\oint_{T'} \vec{u}^{(0)}\cdot\hat{s}\,dl + \alpha\oint_{T'} Q^H\,\vec{u}^{(0)}\cdot\hat{n}\,dl - \oint_{T'} \vec{f}\cdot\hat{s}\,dl$$

$$Q^H \equiv \nabla^2\psi + \beta y$$

$$\equiv \int_{D'} \omega_z^{(1)}\,dA = -\frac{\partial}{\partial t}\int_{D'} (\Gamma^2\sigma\psi_z)_z\,dA + \int_D \Gamma^2\sigma d_z\,dA$$

Stretch T' to T: then require

$$\int_T \vec{u}^{(1)}\cdot\hat{n}\,dl = 0$$

$$\boxed{\frac{\partial}{\partial t}\sum_j \gamma_j - \oint_T \vec{f}\cdot\hat{s}\,dl = 0 \qquad \gamma_j = \int_{C_j} \vec{u}^{(0)}\cdot\hat{s}\,dl}$$

Also valid for each C_j (without $\sum_j$)

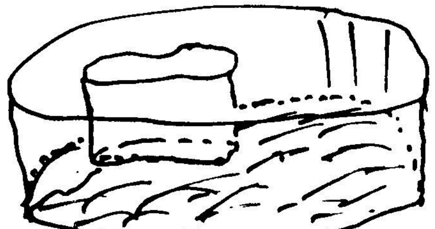
Auxiliary conditions :

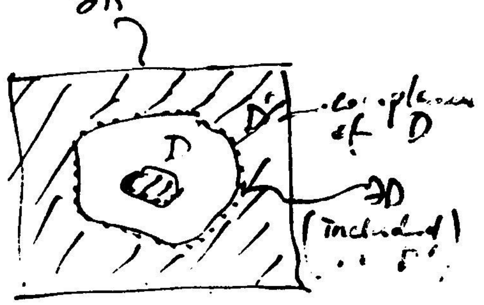
inviscid case

$$\int_D w^{(0)}\,dA = -\Gamma^2\sigma\frac{\partial^2}{\partial t\partial z}\int_D \psi\,dA = 0$$

$$\Rightarrow \quad \boxed{P(z,t) = \int_D \psi\,dA = P(z,0)}$$

INVISCID FLOW - ENCLOSED DOMAINS :





Modal space at time t

$$\boxed{\nabla^2\psi - \lambda^2\psi = f \quad \text{in } R}$$

R = Rectangular Domain

$f|_D = \underline{f_i}$ given

$f|_{D'} = \underline{f_e}_{\text{given}} + \boxed{S_{\partial D}}_{\text{find}} \rightarrow$

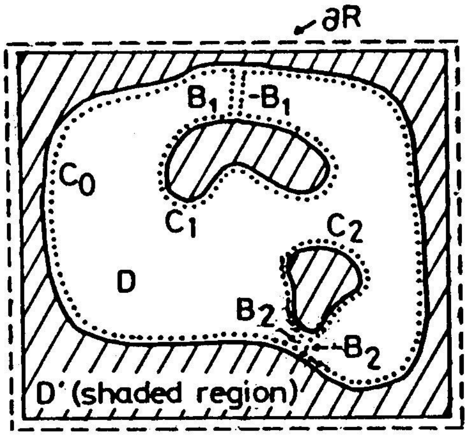
b.c

∂R

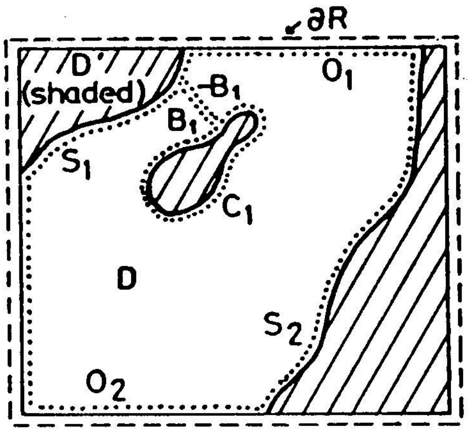
consistency + internal b.c

$\frac{\partial\psi}{\partial s}$ on $\quad j = 0,\dots,N$

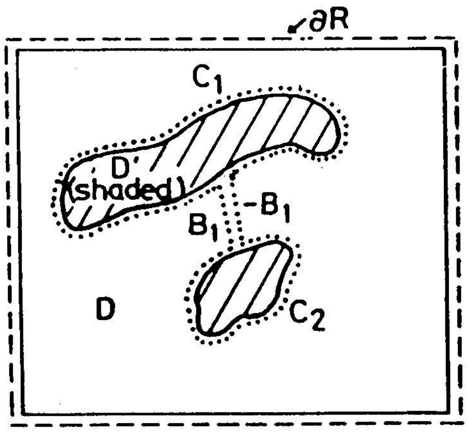
$\int \frac{\partial\psi}{\partial n}\,d\quad = \quad \zeta_j,\ j = 0,\dots$



$$R = D + D'$$
$$\partial D = \Sigma_j \, C_j$$
$$T = \partial D + \Sigma_j \, B_j + \Sigma_j \, (-B_j)$$



$$R = D + D'$$
$$\partial D = \Sigma_j \, C_j + \Sigma_j \, S_j + \Sigma_j \, O_j$$
$$T = \partial D + \Sigma_j \, B_j + \Sigma_j \, (-B_j)$$



$$R = D + D'$$
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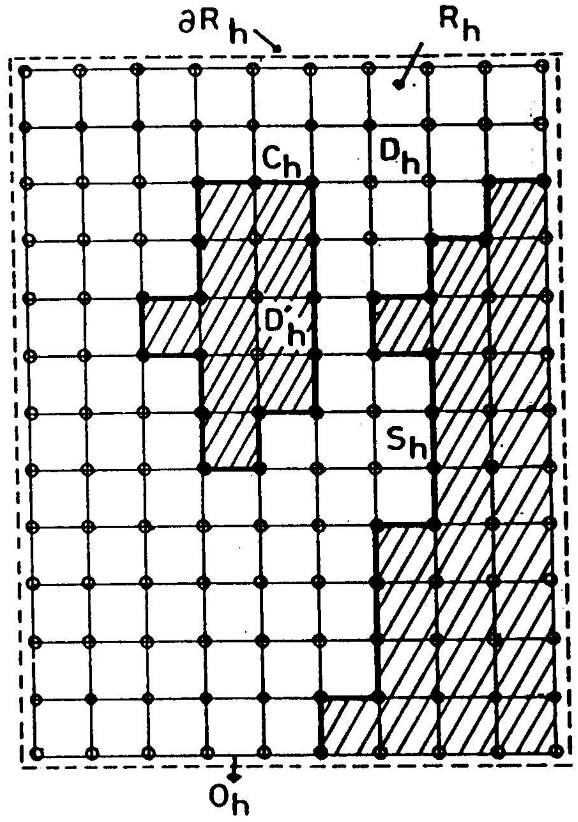
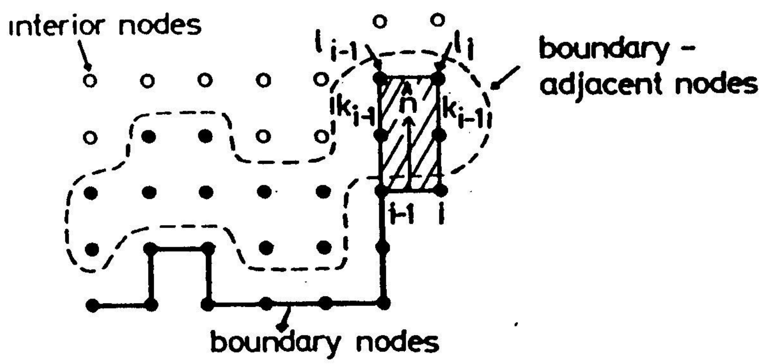
$$T = \partial D + \Sigma_j \, B_j + \Sigma_j \, (-B_j)$$



∂R_h
R_h
C_h
D_h
D'_h
S_h
O_h



interior nodes
l_{i-1}
l_i
boundary - adjacent nodes
k_{i-1}
k_{i-1}
n
i-1
i
boundary nodes

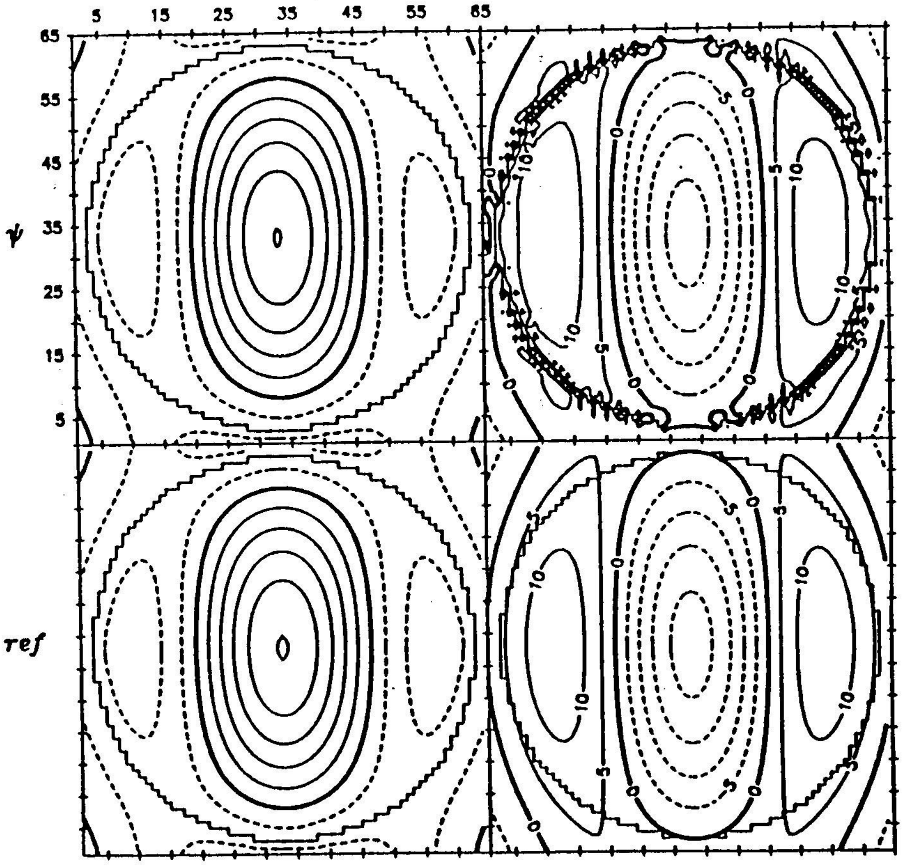
CLOSED BASIN

MILLIFF (1989)

48.7

ROSSBY WAVE BASIN MODES

$t=T/2$

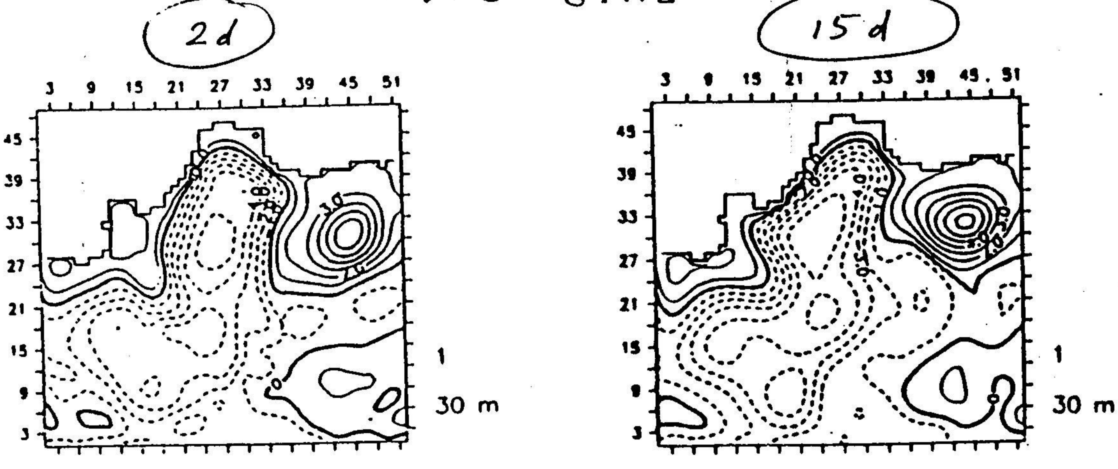


QG Model

Analytical Solution Flierl (1977

INITIALIZATION W/ E. MED DATA

RHODES GYRE



FLIERL (1977)

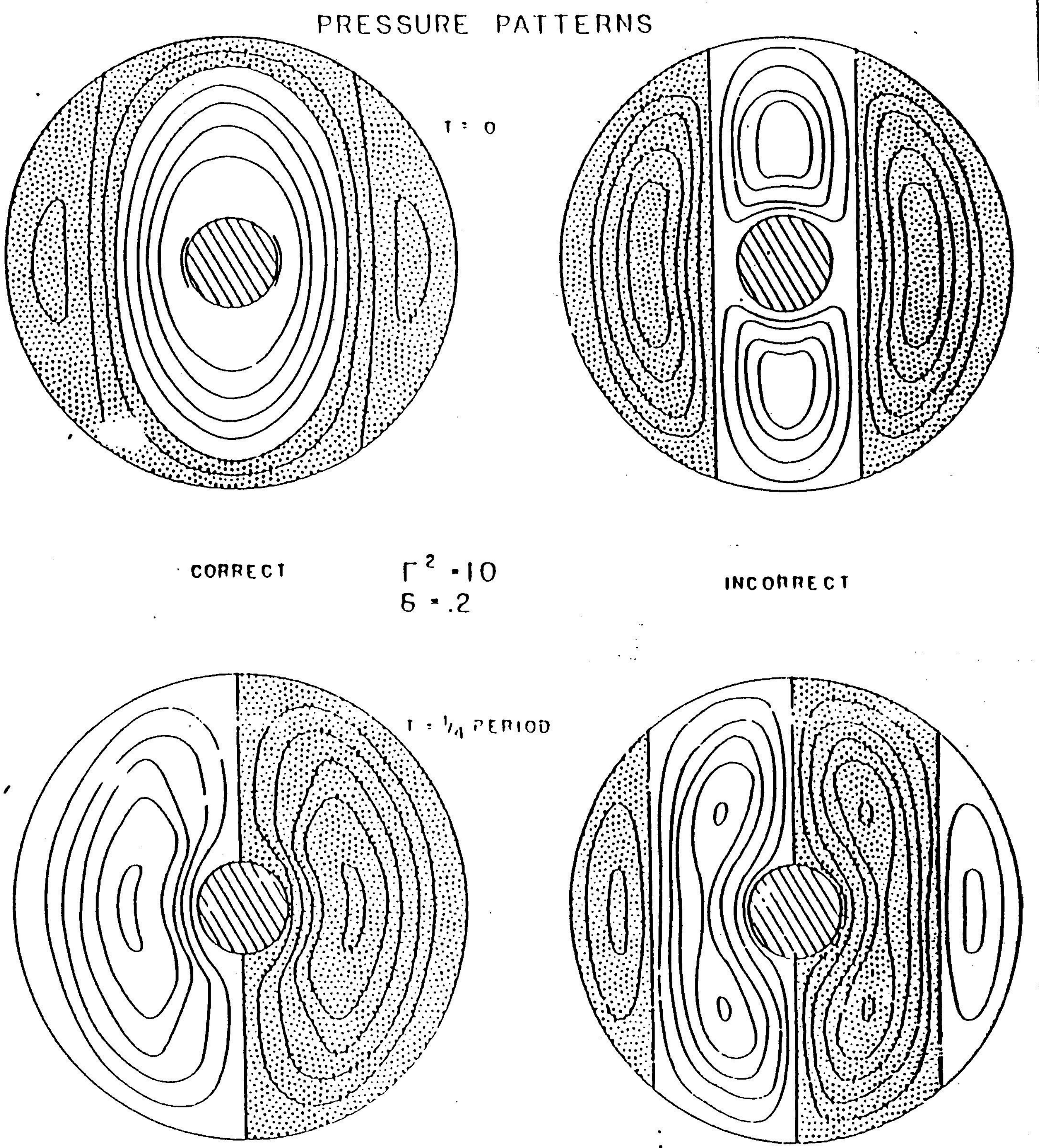
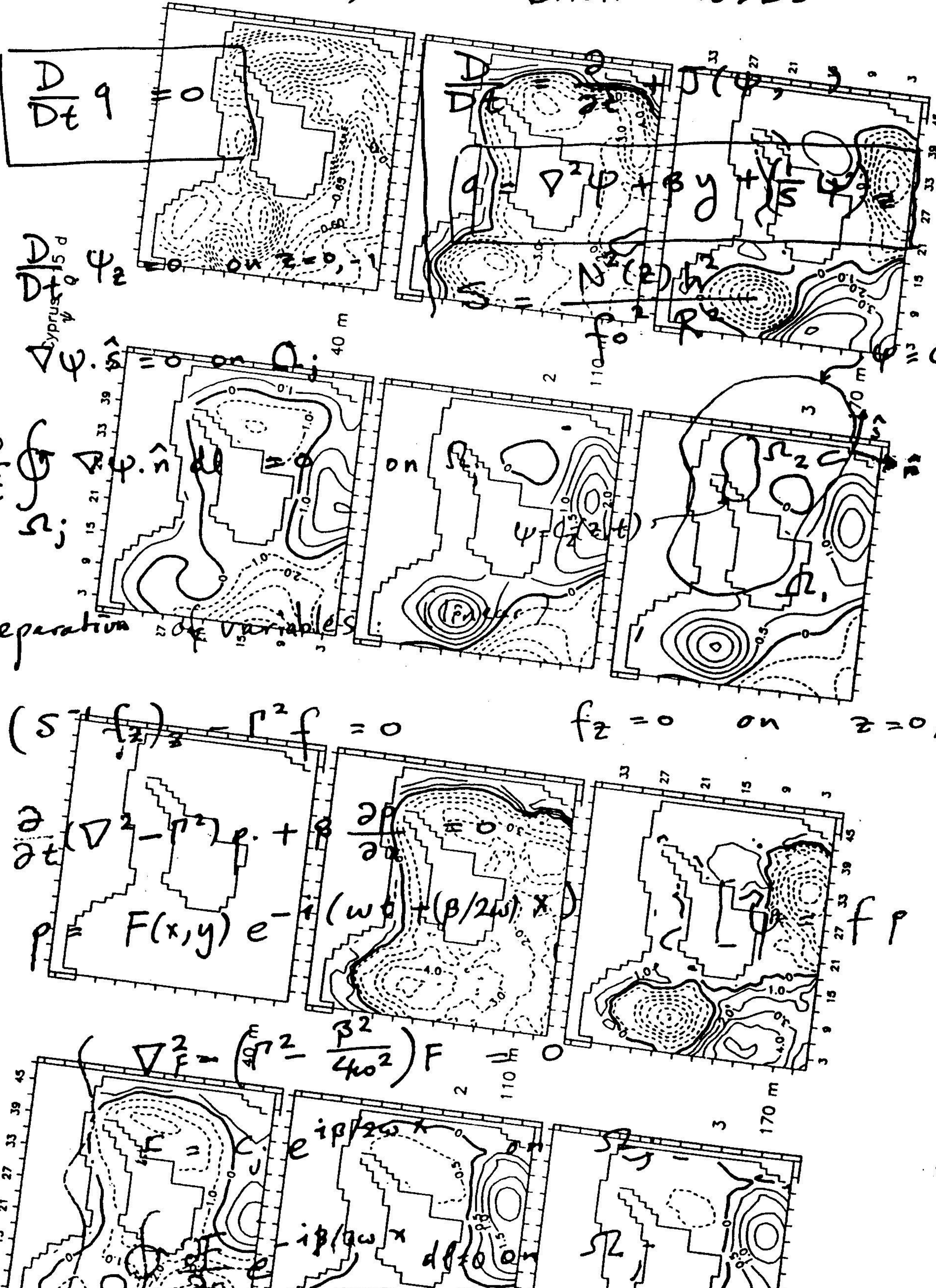


Fig. 4. Correct solutions (left) and incorrect solutions (right) for $\Gamma^2 = 10$ and $\delta = 0.2$ (gravest mode). The solutions are illustrated for two times separated by a quarter period. The maximum p value at $t = 0$ is reached a small distance away from the island, as shown by the small lines near the island on the left and right.

MULTIPLY CONNECTED (CLOSED) OCEAN DOMAINS

FLIERL (1977): BASIN MODES



Vorticity eqn $\boxed{\frac{D}{Dt} q = 0}$ $\frac{D}{Dt} = \frac{\partial}{\partial t} + J(\psi, \;)$

$$q = \nabla^2\psi + \beta y + \left(\frac{1}{S}\psi_z\right)_z$$

$$S = \frac{N^2(z)\, h^2}{f_0^2 R^2}$$

b.c. $\frac{D}{Dt}\psi_z = 0$ on $z = 0, -1$

$\nabla\psi\cdot\hat{s} = 0$ on Ω_j $\psi = C_1(z,t)$

$$\frac{\partial}{\partial t}\oint_{\Omega_j} \nabla\psi\cdot\hat{n}\, dl = 0 \quad \text{on } \Omega_j$$

$\psi = C_2(z,t)$

Separation of variables:

$(S^{-1} f_z)_z - \Gamma^2 f = 0$ $f_z = 0$ on $z = 0, -1$

$$\frac{\partial}{\partial t}(\nabla^2 - \Gamma^2)p + \beta \frac{\partial p}{\partial x} = 0$$

Let $p = F(x,y)\, e^{-i(\omega t + (\beta/2\omega)x)}$ $[\psi = f p]$

$$\nabla^2 F - \left(\Gamma^2 - \frac{\beta^2}{4\omega^2}\right)F = 0$$

$F = C_j e^{i\beta/2\omega\, x}$ on Ω_j

$$\oint \frac{\partial F}{\partial n} e^{-i\beta/2\omega\, x}\, dl = 0 \quad \text{on } \Omega_j$$

48.10

FLIERL (1977) Analytic Solution -- Two Levels

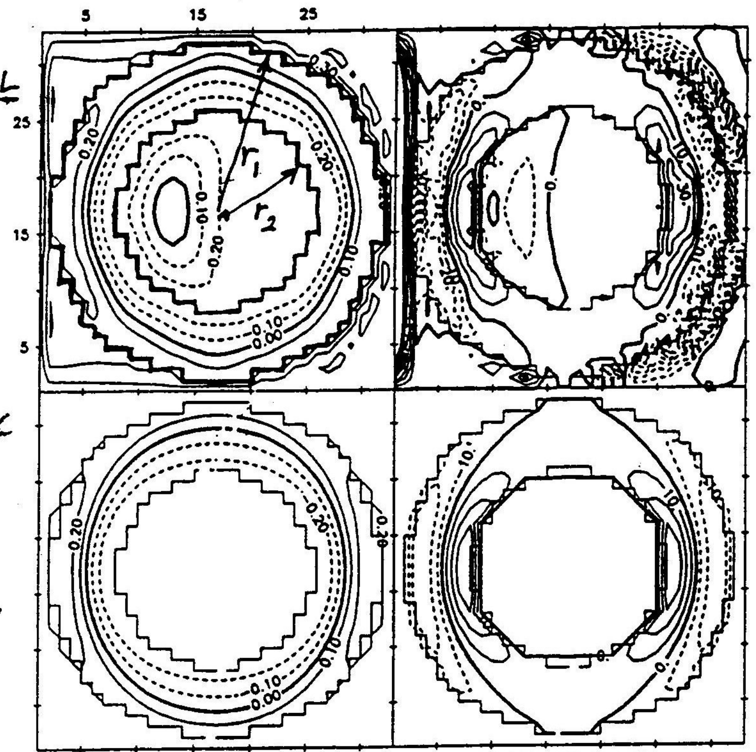
$r_1 = 1.$

$r_2 = 0.6$ $\Gamma^2 = 10$

gamsq=10,r=0.6,lowres,t=T

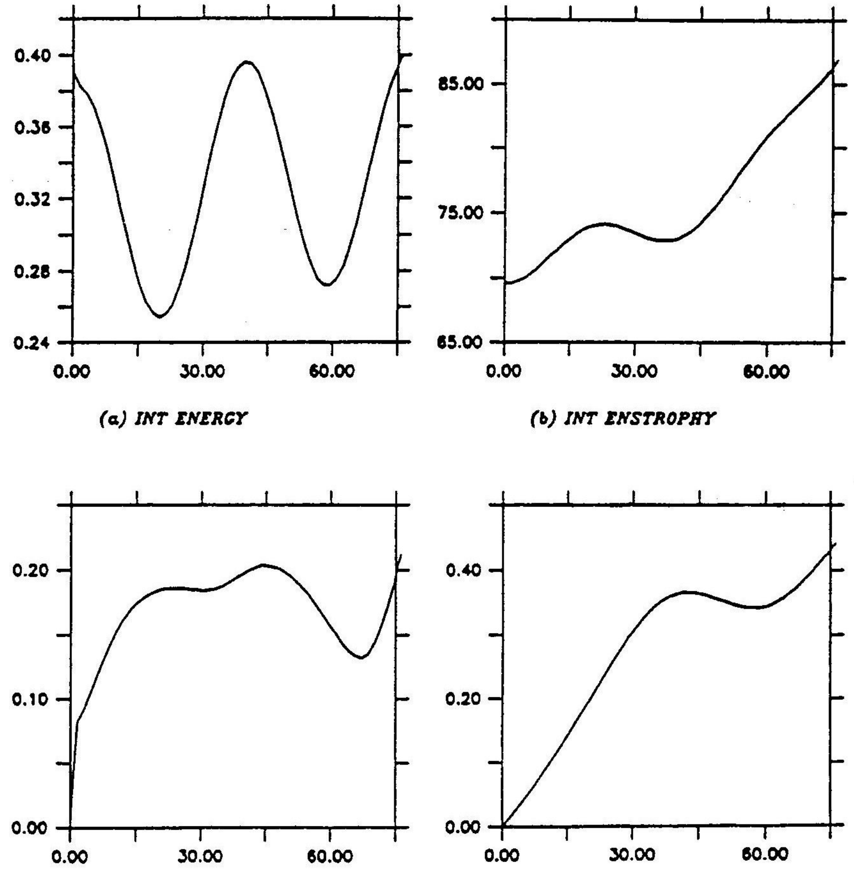
MODEL

ANALYTIC SOLN.



STREAMFUNCTION VORTICITY

33 x 33 resolution



(a) INT ENERGY

(b) INT ENSTROPHY

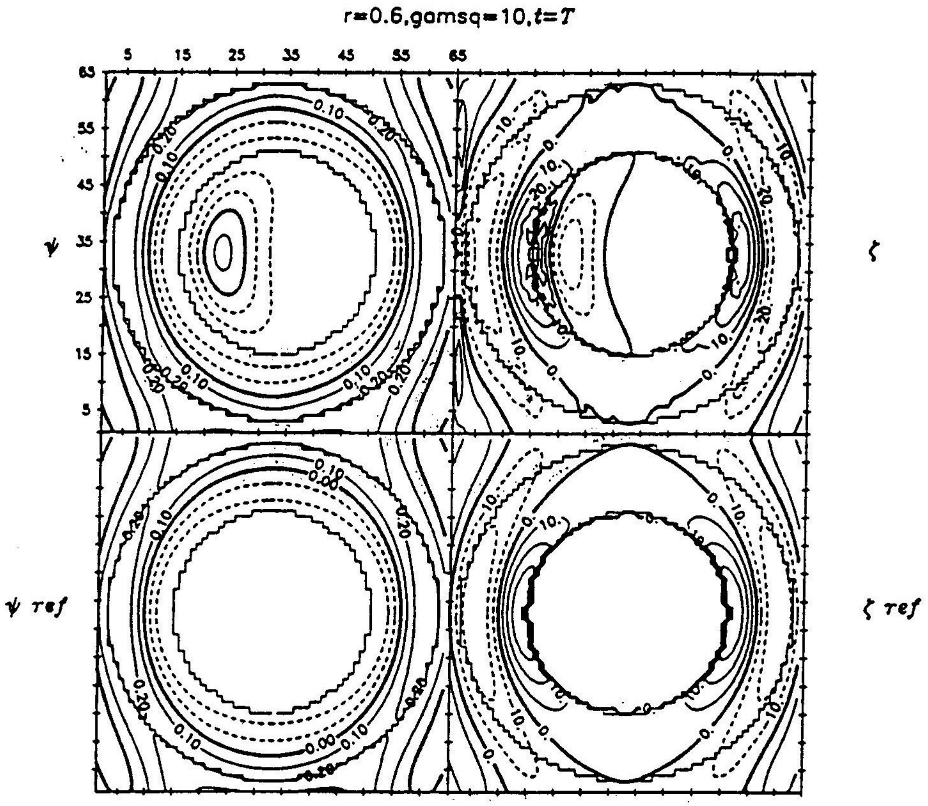
(c) ψ_{rms} ERROR

(d) ζ_{rms} ERROR

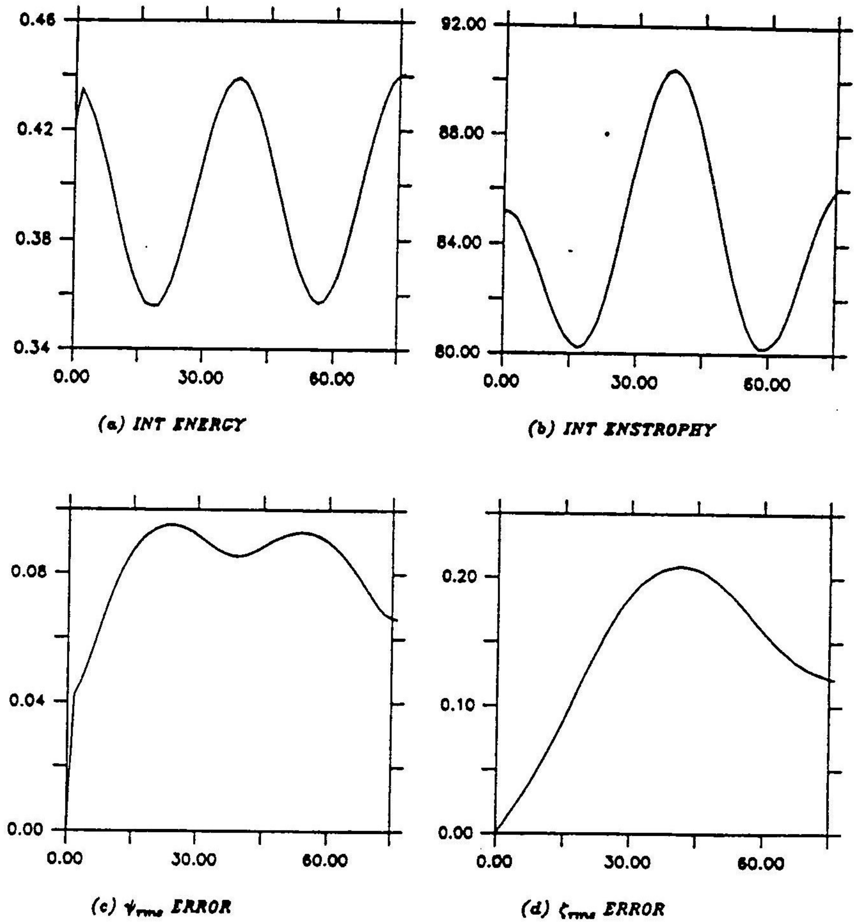
48.11

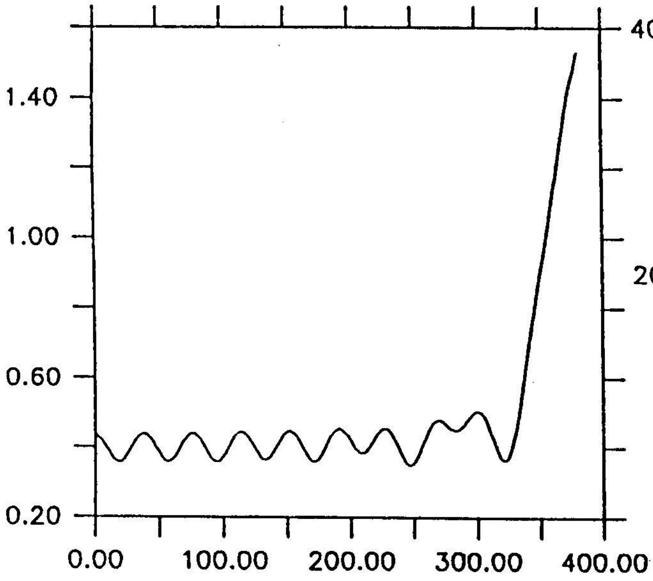
$r_1 = 1$

$r_2 = 0.6$ $\qquad \Gamma^2 = 10$

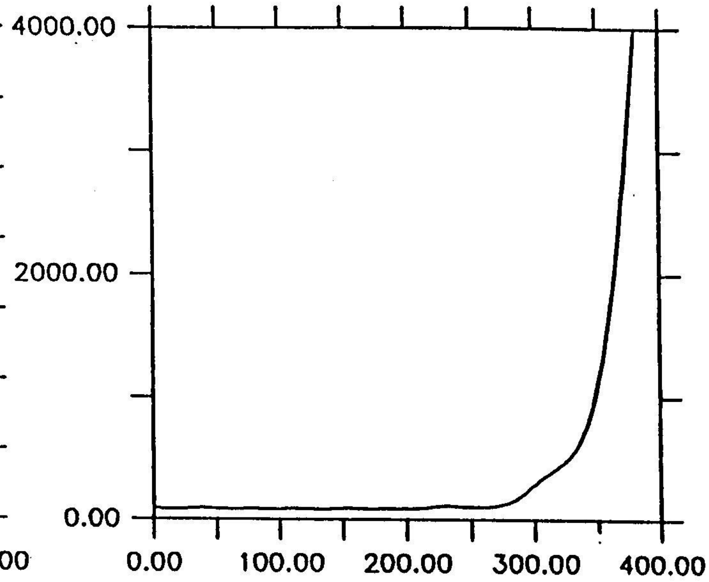


65×65

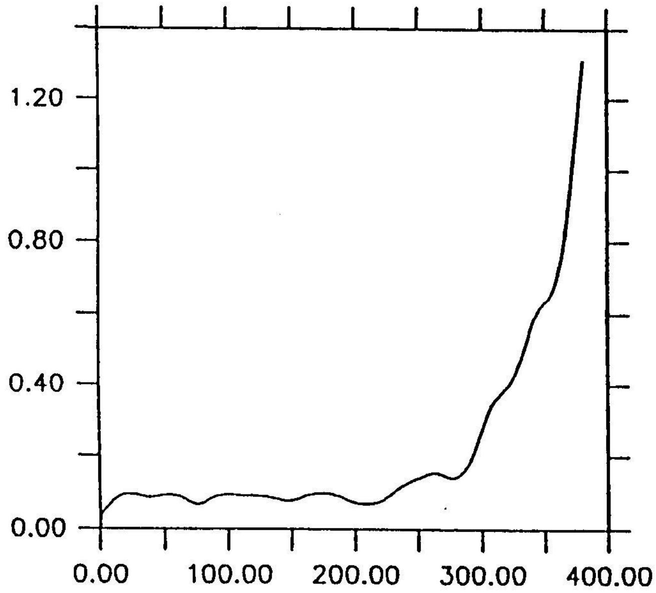




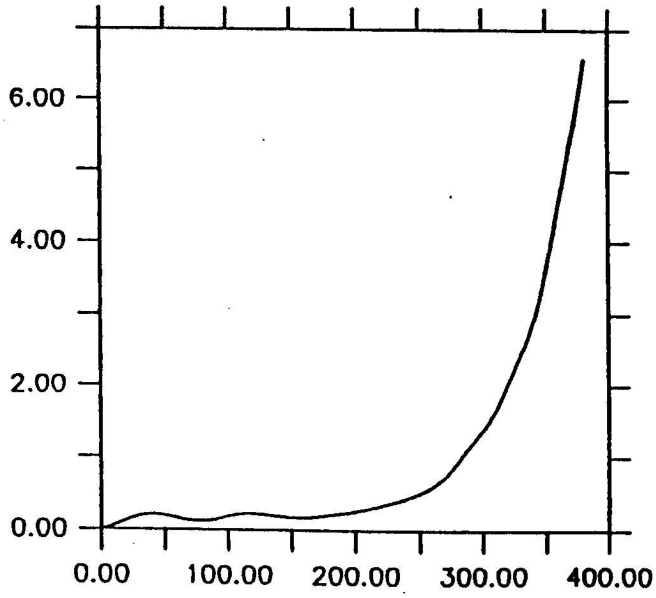
(a) INT ENERGY



(b) INT ENSTROPHY



(c) ψ_{rms} ERROR



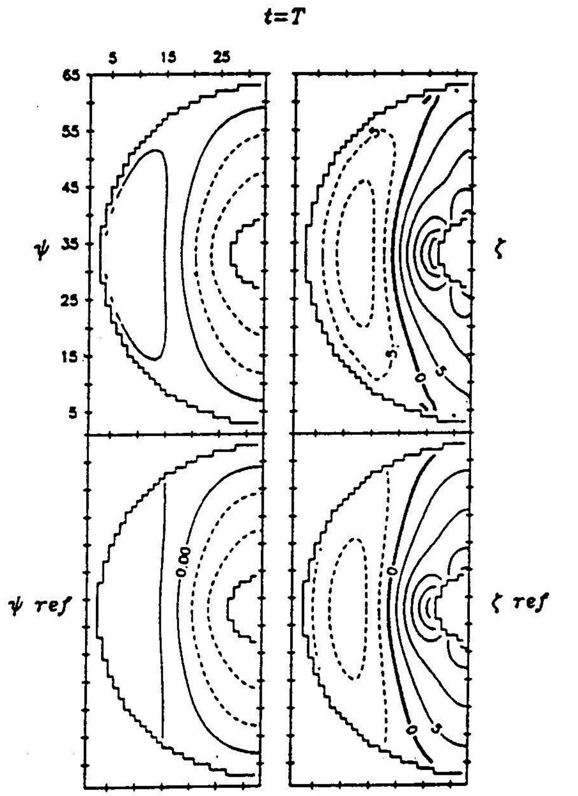
(d) ζ_{rms} ERROR

48.13

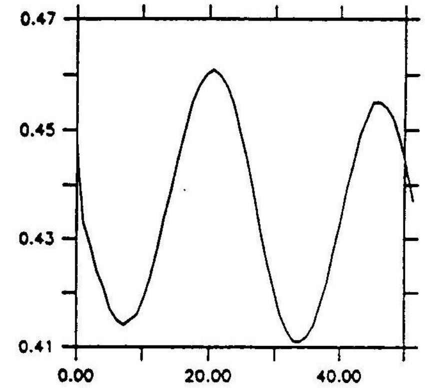
SEMI-OPEN DOMAIN

$r_1 = 1$ $\quad \Gamma^2 = 10$

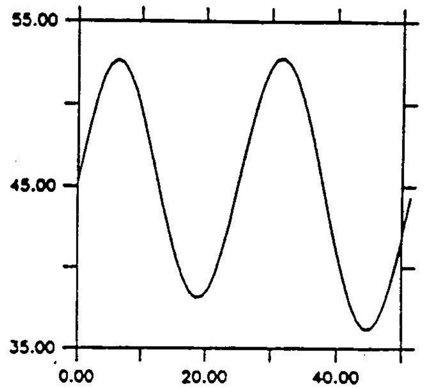
$r_2 = 0.2$



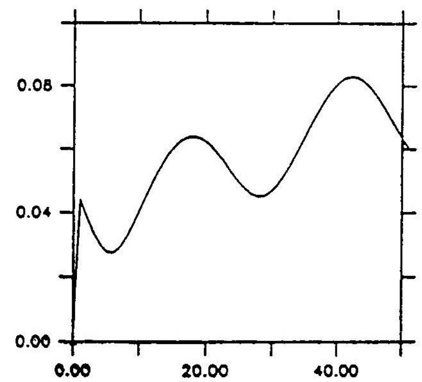
SCENT: Multiply connected domain w/ open boun



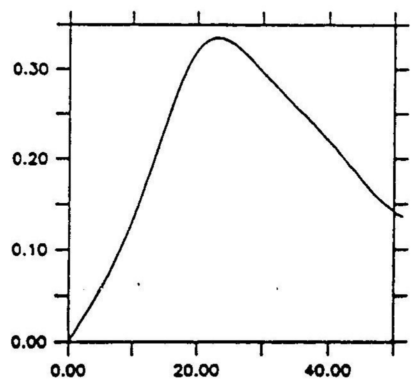
(a) INT ENERGY



(b) INT ENSTROPHY



(c) ψ_{rms} ERROR



(d) ζ_{rms} ERROR

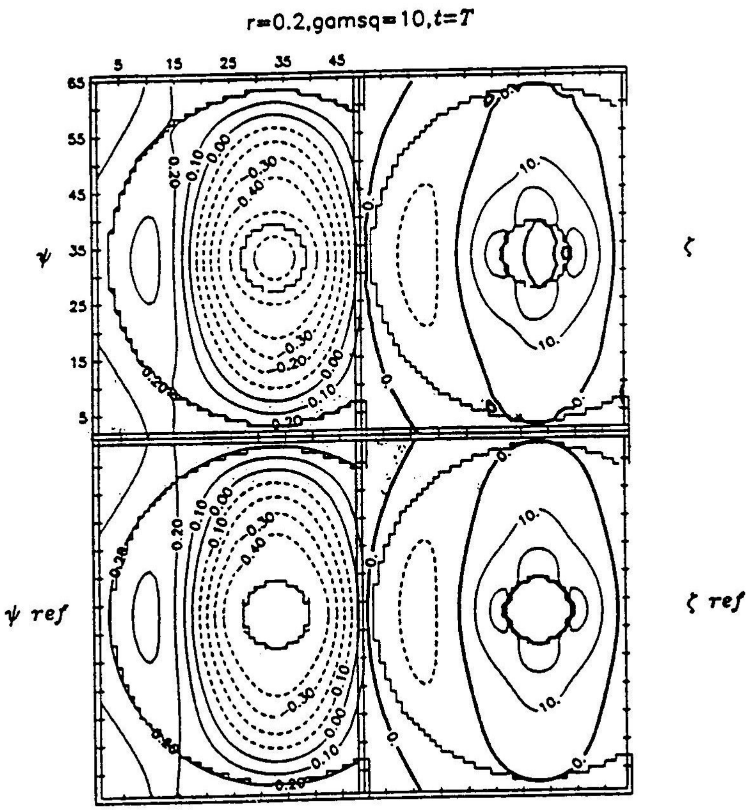
48.14

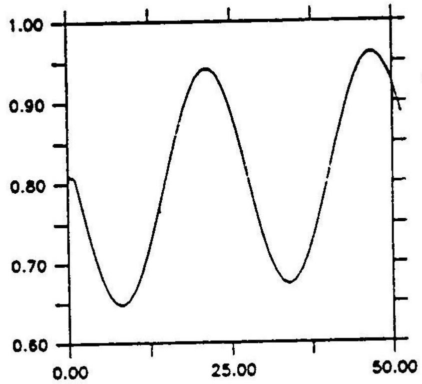
SEMI-OPEN DOMAIN

$r_1 = 1$

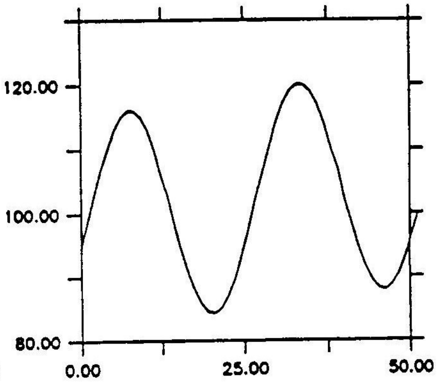
$\Gamma^2 = 10$

$r_2 = 0.2$

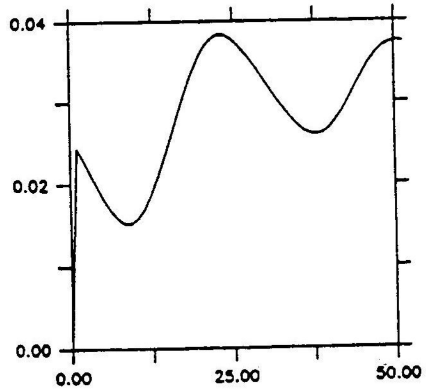




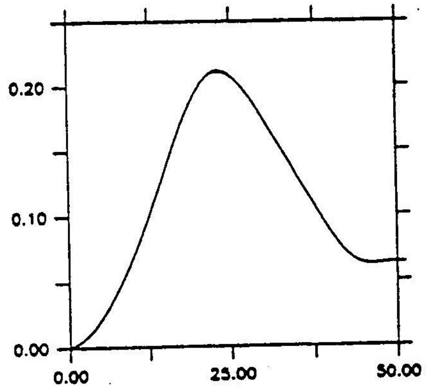
(a) INT ENERGY



(b) INT ENSTROPHY



(c) ψ_{rms} ERROR

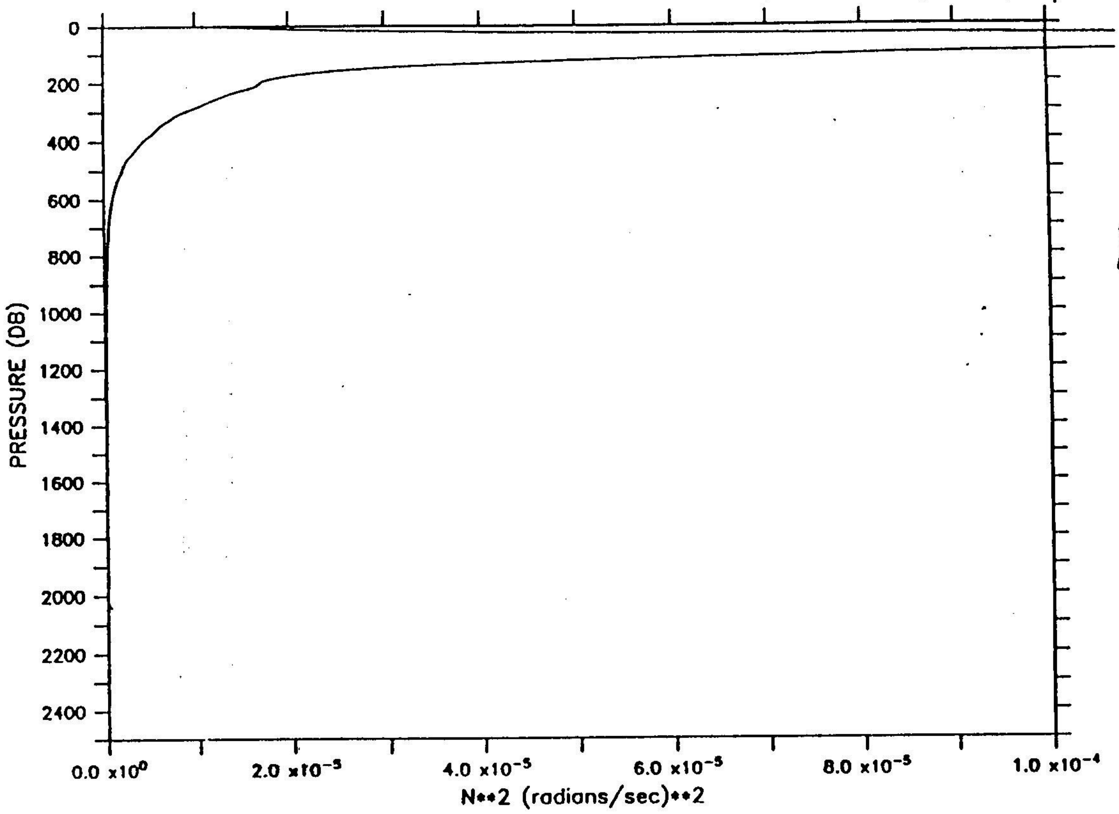


(d) ζ_{rms} ERROR

LEVANTINE BASIN – EASTERN MEDITERRANEAN

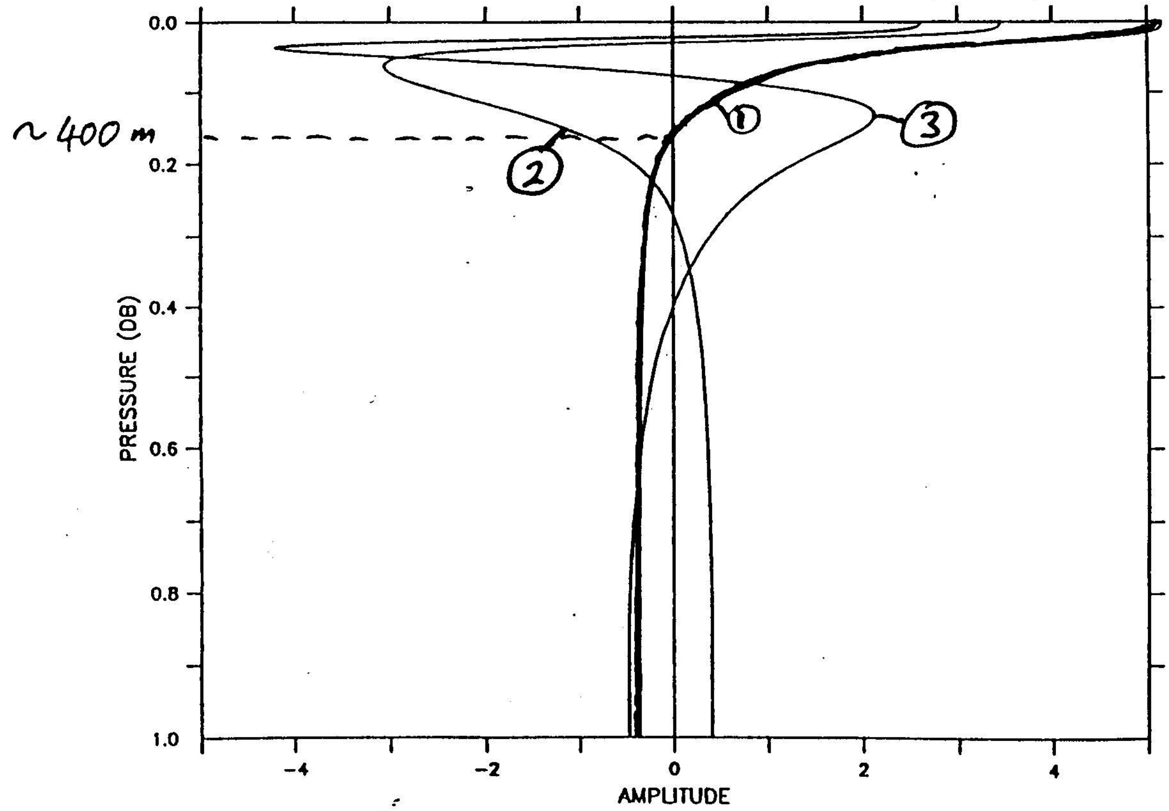
BRUNT-VAISALA (STABILITY) FREQUENCY

POEM1

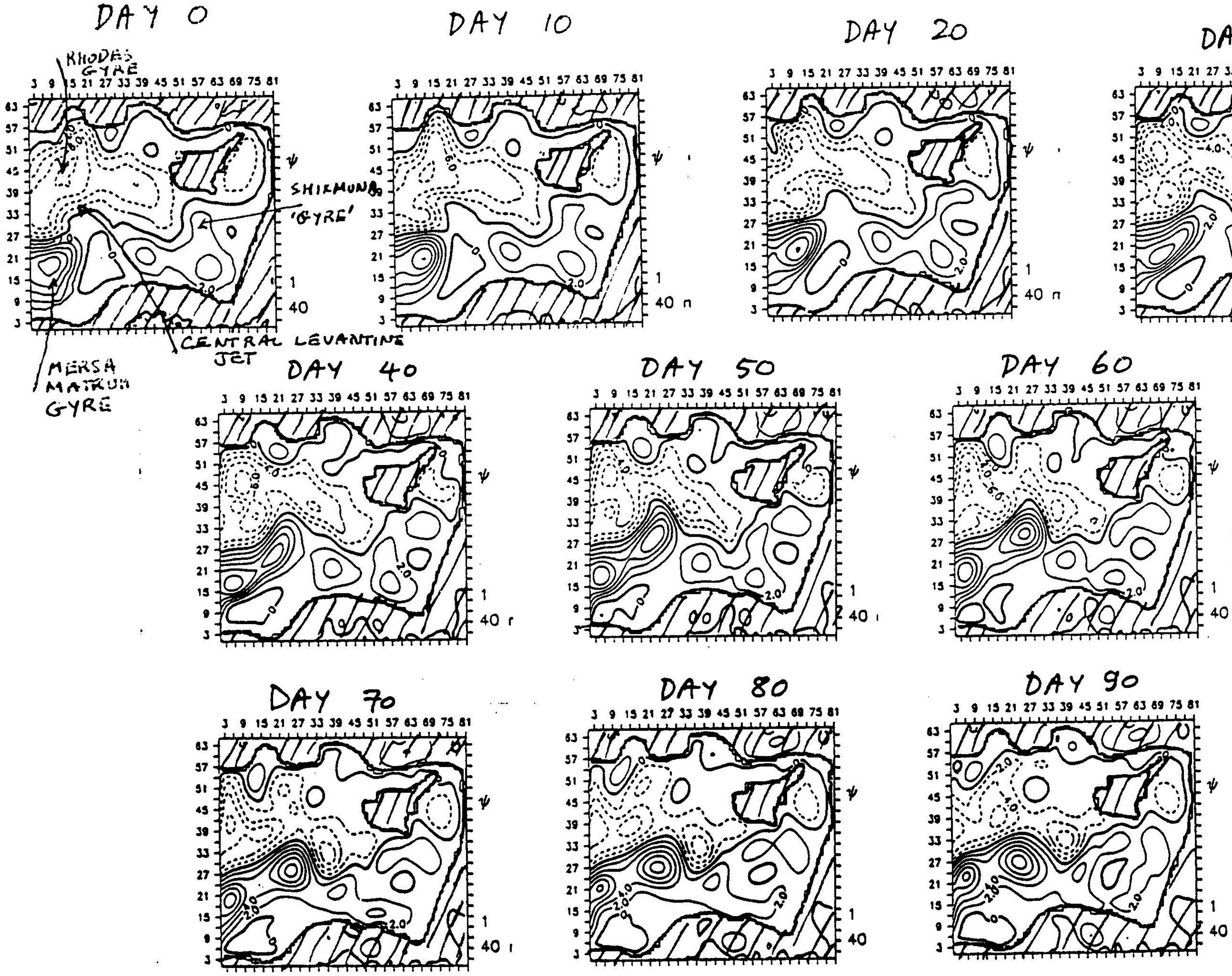


N**2 for levantine sea – poem1

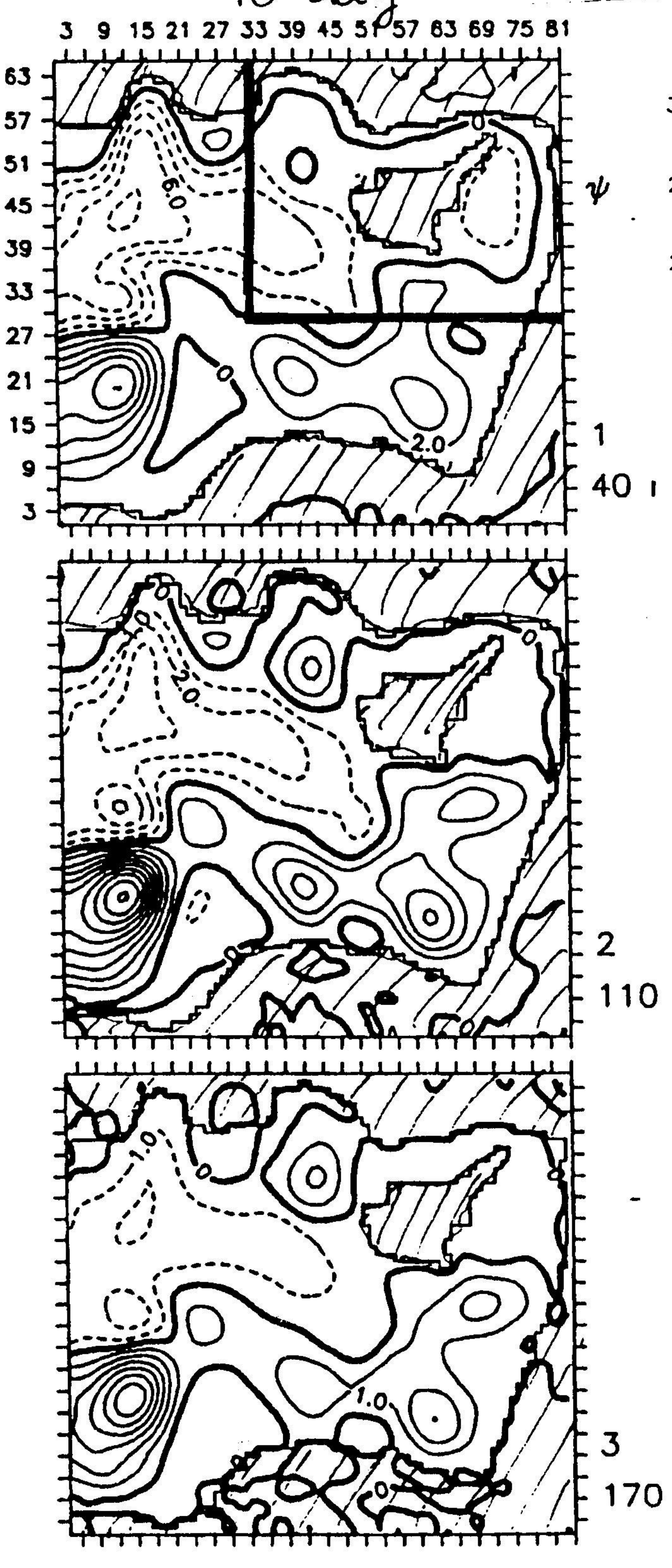
VERTICAL STRUCTURE – EIGENMODES

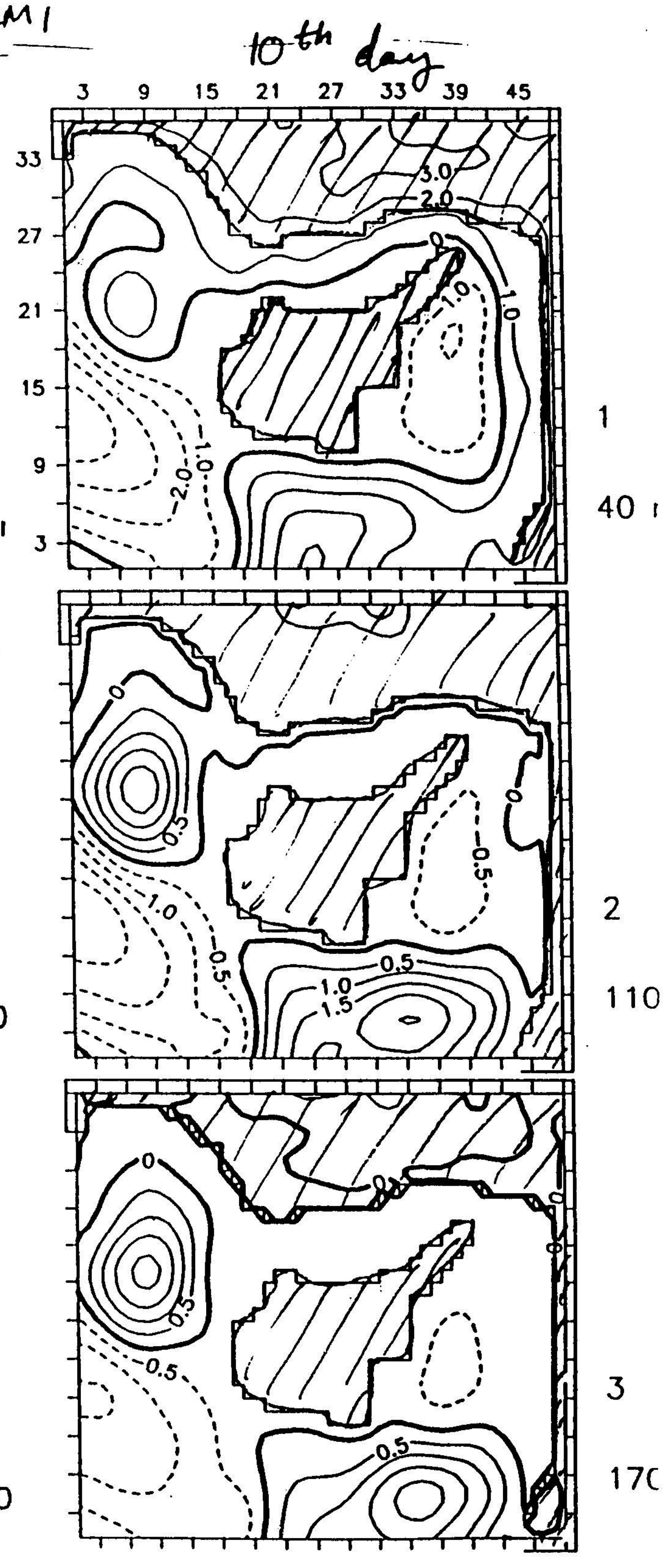


POEM1 COASTAL QG MODEL RUN



INITIALIZATION
POEMI

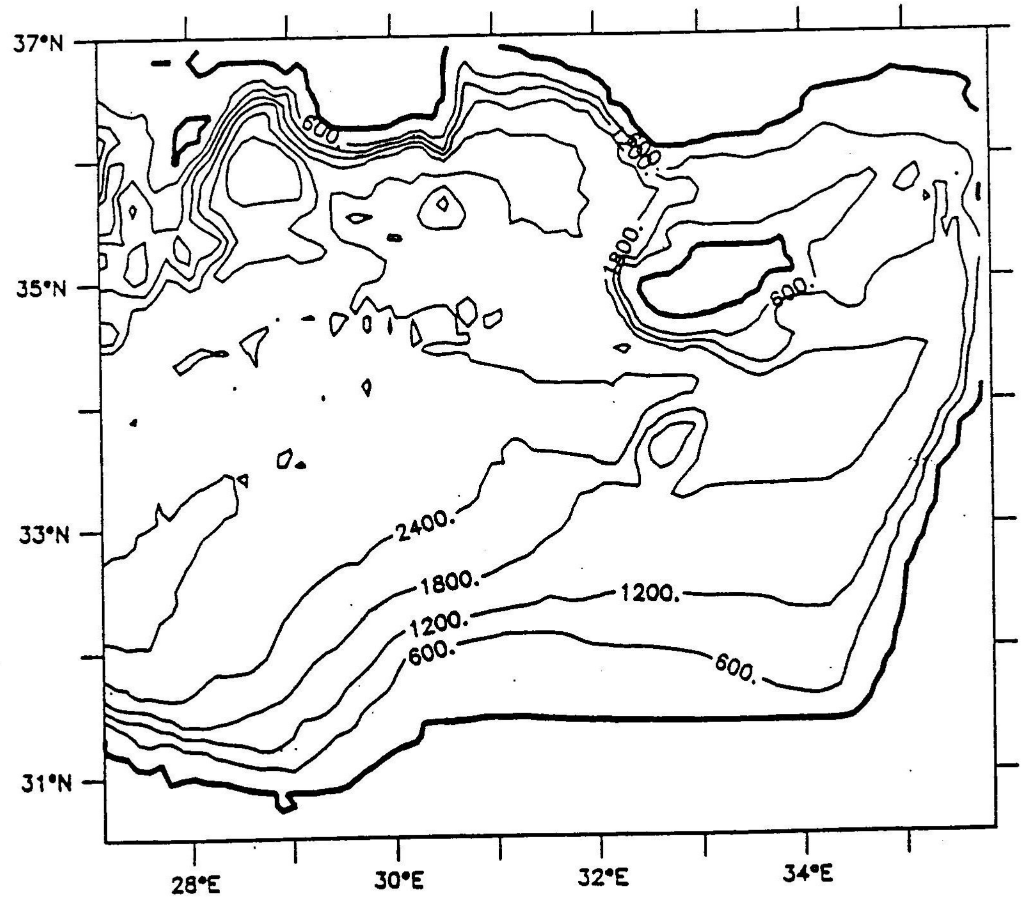


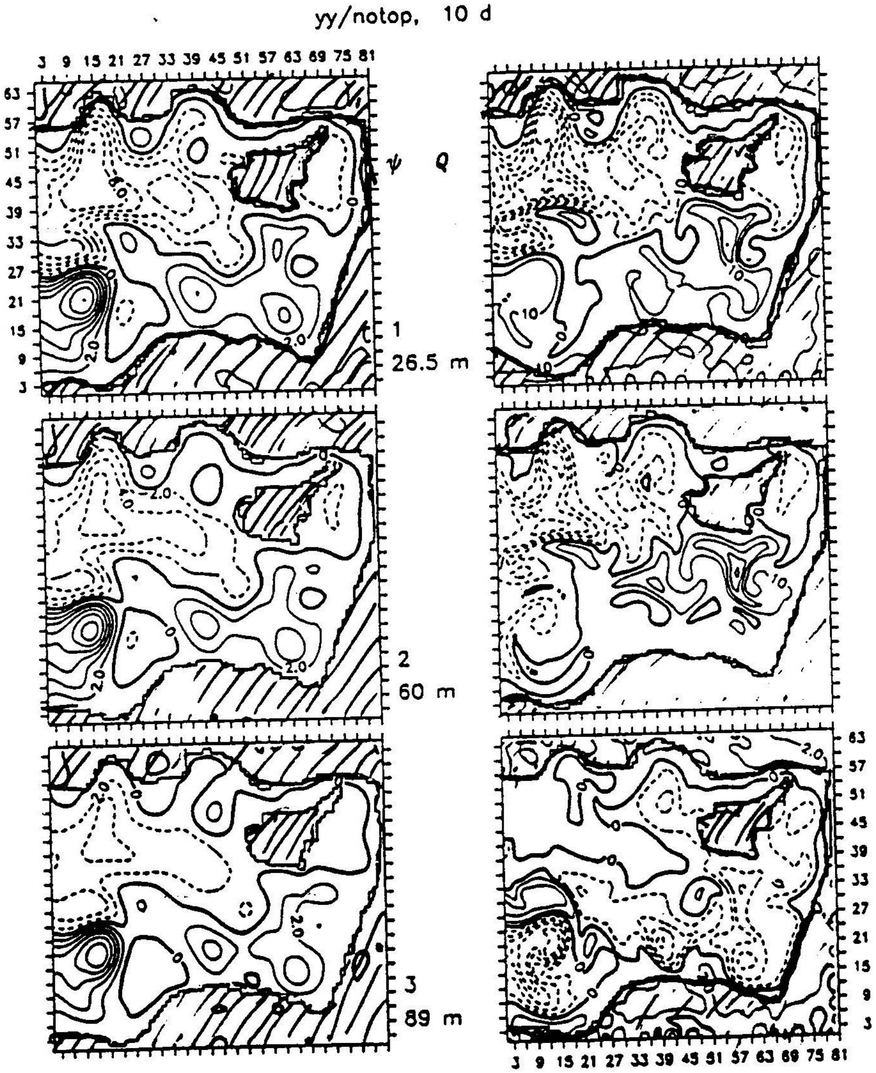


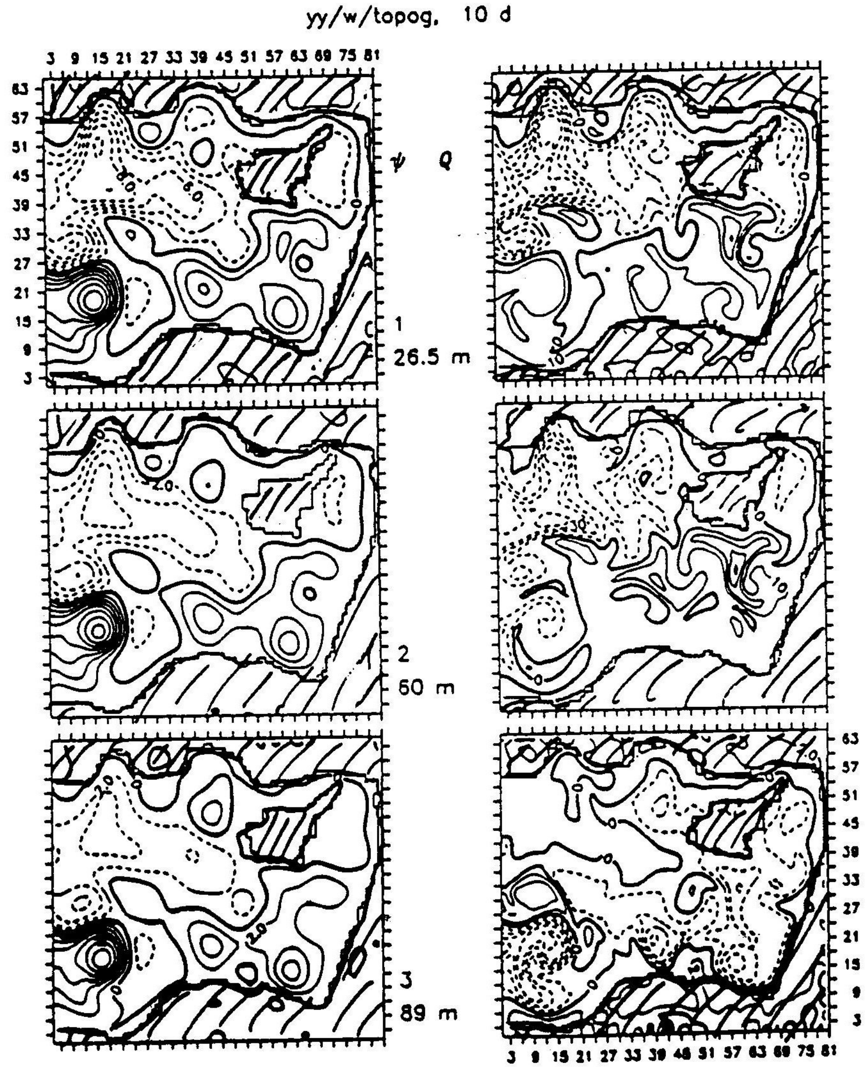
Contour interval 1.0

Contour interval 2.0

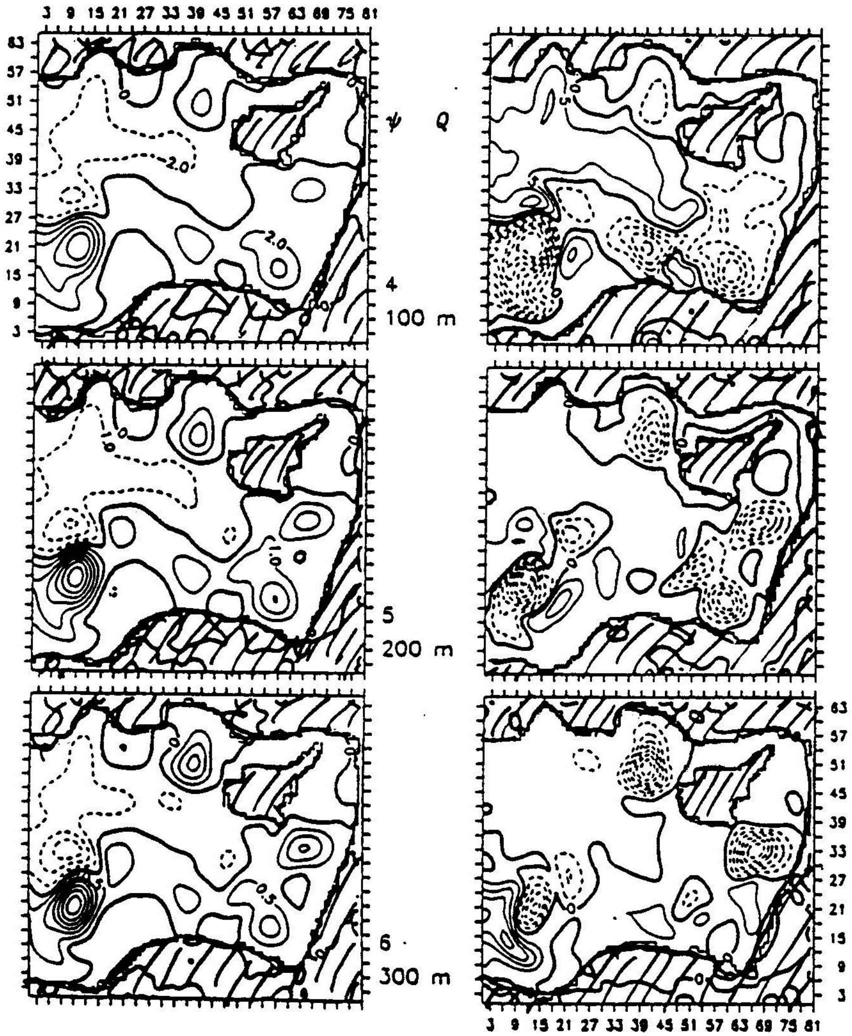
$\Delta x = \Delta y = 10$ km


37°N
35°N
33°N
31°N
28°E
30°E
32°E
34°E
2400.
1800.
1200.
600.
1200.
600.
1800.
600.
600.

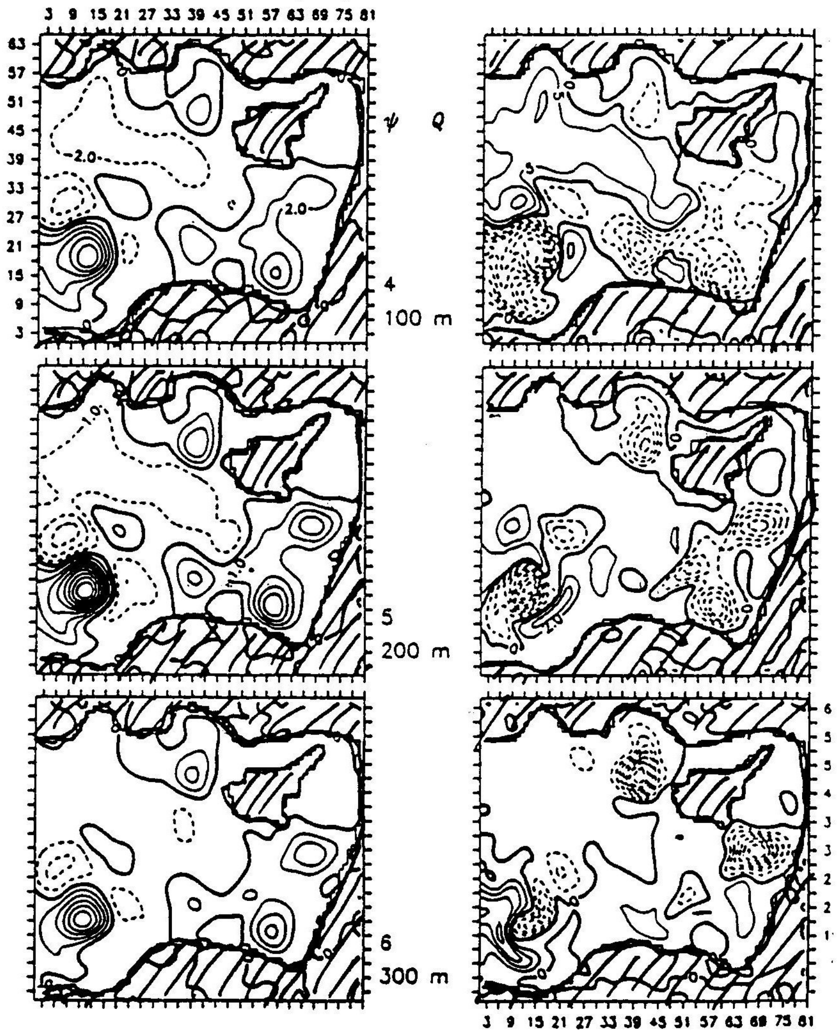

yy/notop, 10 d
3 9 15 21 27 33 39 45 51 57 63 69 75 81
63 57 51 45 39 33 27 21 15 9 3
ψ Q
1
26.5 m
2
60 m
3
89 m


yy/w/topog, 10 d
3 9 15 21 27 33 39 45 51 57 63 69 75 81
63 57 51 45 39 33 27 21 15 9 3
ψ Q
1
26.5 m
2
60 m
3
89 m

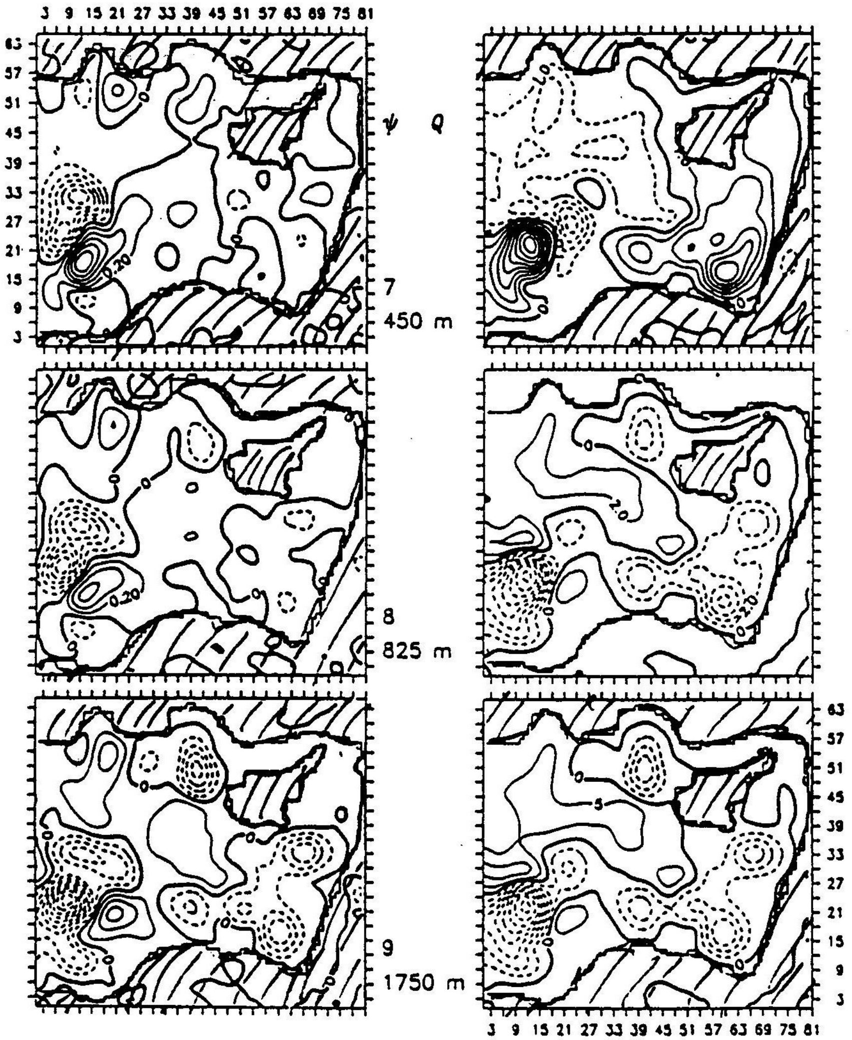
yy/notop, 10 d



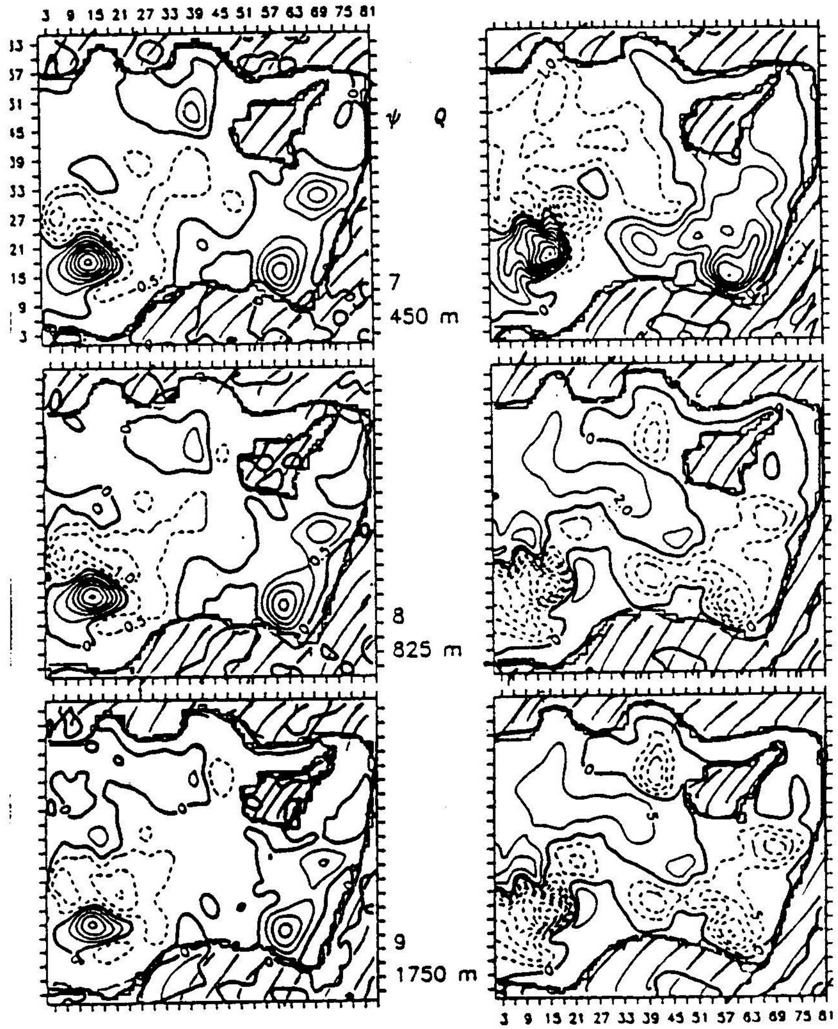
yy/w/topog, 10 d



yy/notop, 10 d



yy/w/topog, 10 d

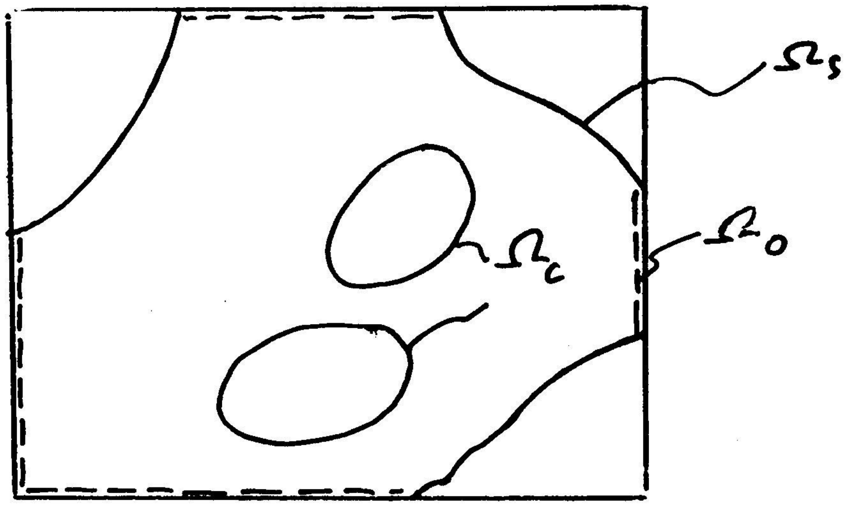


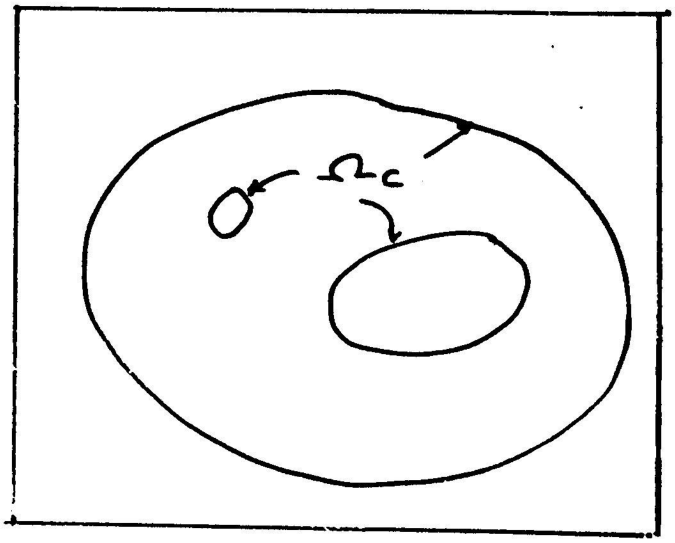
Ω_0 CFvN b.c.

Ω_s $\psi = g(z,t)$ specified

Ω_c * $[\psi = g(z,t)]$: $\frac{\partial \psi}{\partial s} = 0$

* $\frac{\partial}{\partial t}\int_{\Omega_j} \frac{\partial \psi}{\partial n}\, ds = 0$





FOURTH POEM SCIENTIFIC WORKSHOP

Istituto Studio Dynamica Grandi Masse–CNR
and
Istituto Veneto di Scienze, Lettre ed Arti
Venice, Italy

27 August — 1 September 1990